

DC/PAK/77/039/ED
Strengthening Selected Educational
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C U R R I C U L U M D E V E L O P M E N T

A SERIES OF PAPERS ON THE SUBJECT WITH
SPECIAL REFERENCE TO N.W.F.P. PAKISTAN

BY

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THESE PAPERS WERE PRODUCED FOR THE BUREAU OF CURRICULUM DEVELOPMENT, ABBOTTABAD, PAKISTAN WITH THE ASSISTANCE OF UNESCO PROJECT 77/039, STRENGTHENING SELECTED EDUCATIONAL INSTITUTIONS IN N.W.F.P.

THEY ARE INTENDED FOR THE USE OF INSTRUCTORS IN THE PRIMARY TEACHER TRAINING INSTITUTIONS AND FOR THE MEMBERS OF THE TASK FORCES AND OTHER GROUPS PREPARING MATERIAL FOR THE PRE-SERVICE AND IN-SERVICE TEACHER TRAINING.

THE AUTHOR WISHES TO EXPRESS HIS THANKS TO PROFESSOR HABIB UR RAHMAN, CHAIRMAN OF THE N.W.F.P. TEXTBOOK BOARD FOR HIS MAJOR ASSISTANCE IN THE PREPARATION OF THE MATERIAL.

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THE VIEWS EXPRESSED IN THIS PAPER ARE THOSE OF THE AUTHOR AND ARE NOT NECESSARILY THOSE OF UNESCO OR THE DEPARTMENT OF EDUCATION, N.W.F.P.

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# C U R R I C U L U M

## INTRODUCTION

Curriculum deals with the issue, "What shall we teach?". It should be kept in mind, however, that the way things are taught and the structure set up for the teaching are all part of the total curriculum.

## ALTERNATIVES

It is not possible to teach a child everything. No-one can teach everything or learn everything. A severe process of selection must take place and so priorities must be established.

The problem is rarely that of choosing subject matter. In many ways this tends to choose itself. The problem comes in the handling of the material. What interpretation is to be put onto it? A curriculum is "a reflection of what people think, feel, believe and do".<sup>1</sup> The focus of a curriculum is influenced by the critical events of the times as well as the culture in which it is imbedded.

## PROCESS

The actual process of curriculum design is pretty well the same the world over. Goals are established by a certain group of people and then a curriculum is prepared in an effort to reach those goals.

Tyler<sup>2</sup> proposed that the following four questions should be asked in this order:

1. *What educational purposes should the school seek to attain?*
2. *What educational experiences can be provided that are likely to attain these purposes?*
3. *How can these educational experiences be effectively organised?*
4. *How can we determine whether these purposes are being attained?*

Despite the strong arguments that beginning curriculum development by defining objectives has a stultifying effect on the procedures, this approach has been in use for some 60 years, before and after Tyler, and will be used well into the future. It is THE method.

Its main weakness is that it emphasises measurable overt behaviour and much of a teacher's work is to try to develop patterns of thinking which are not always susceptible of direct measurement.

## CRITERIA

In order to make a rational and systematic selection between alternatives and to avoid unproductive arguments, criteria must be established. Here are some possibilities:<sup>3</sup>

1. The curriculum content should reflect contemporary scientific knowledge.
2. The more fundamental the knowledge, the stronger the reason for its inclusion.
3. It must be in tune with the social and cultural realities of the time.  
It must have significance.
4. There must be a balance between breadth and depth.
5. There must be the acquisition of significant new knowledge and the development of increasingly more effective ways of thinking, desirable attitudes and interests, and appropriate habits and skills.
6. It should be appropriate to the needs and interests of the learners.

7. It should be learnable. That is, it should be adjusted to the ability of the learners and the social heritage has to be translated into experiences which enable each pupil to make it his own.

## Focus

If we look at each of the decades since 1920 we can see some of the foci that were products of their time:

|        |                                                        |
|--------|--------------------------------------------------------|
| 1920's | Subject-centred                                        |
| 1930's | Child-centred                                          |
| 1940's | Society-centred                                        |
| 1950's | Emphasis on mental health and individual excellence    |
| 1960's | Emphasis on the culturally disadvantaged. <sup>4</sup> |

## QUESTION

*"Is it possible in the North West Frontier Province of Pakistan in the 1980's to identify a common focus to which the whole curriculum is geared?"*

Should this focus be:

- \* the subject disciplines
- \* the Pakistani child
- \* Pakistani society
- \* the disadvantaged in society
- \* the modern Islamic state?

*or what?*

~~~~~

T.L.M. oct 81

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DISCUSSION POINTS ARISING FROM THE PAPER

1. The first of the four questions delineated by Tyler is redundant at the level of curriculum development in N.W.F.P. The purposes relevant to the N.W.F.P. circumstances have already been defined. Teachers will not be called upon individually to decide WHAT SHOULD WE TEACH? This status has to be accepted.
2. The next two questions are of real worth to us.

WHAT EDUCATIONAL EXPERIENCES CAN BE PROVIDED THAT ARE LIKELY TO ATTAIN THESE PURPOSES?

HOW CAN THESE EDUCATIONAL EXPERIENCES BE EFFECTIVELY ORGANISED?

Both are intimately related to methodology and are answered by individual teachers in a highly inadequate manner. Both pertain to what should go on inside a classroom. It is in this area that our underdevelopment is at its peak. It is here that we are behind the international mainstream by about 50 - 60 years.

Specifically we must ensure that:

- 2.1 the existing one-way interaction inside a classroom is modified to a two-way interaction
- 2.2 the role of a teacher as dispenser of knowledge is replaced by the role as a facilitator of learning
- 2.3 the imposed type of teaching undertaken now gives way to meaningful learning on the part of the kids
- 2.4 the rote-type of learning based on excessive drill is substituted by the provision of experiences to kids to foster learning through a guided discovery approach
- 2.5 the stress on textbooks as the sole source of knowledge is de-emphasised and shifted towards the usage of no-cost teaching/learning aids
- 2.6 the existing teaching techniques which cater to mental operations of a low order are replaced by ones securing mental operations of a higher order to ensure better realisation of an individual's potential
- 2.7 the existing practice of conducting a subjective type of test at the end of the year be modified to incorporate objective evaluation conducted throughout the year.

3. Very often in the type of situation that exists in N.W.F.P. it is the second of the above two questions that takes precedence. In other words, given the limited range of organisational possibilities open to us, what educational experiences can be provided?
4. Teachers do not enjoy the autonomy of being able to undertake the curriculum development process from the start. They come in somewhere in the middle of the process and are called upon to translate into practice what others have decided they should accomplish.

The challenge is, relying on the scarce resources at our disposal, to equip this inadequately trained force sufficiently to attain our objectives.

5. The focus of the curriculum is the revival of Islam and at the moment this is being interpreted in various personal ways. A general concensus will need to emerge as regards the politico-socio-economic structure of the Islamic polity.

This process is now well under way and Muslim intellectuals are bending their backs to grapple with the issues but time is needed to sift the chaff from the grain.

6. The one fact that stands out clearly is that this whole region is underdeveloped and the world is divided into developed and developing states. This is not a natural phenomenon to be accepted passively. This curse has been, in some degree in an unconscious manner, a human imposed affair. We shall have to get rid of it if decency is to prevail everywhere.

Eradication of this plague is unthinkable without providing for some human resources along with other appropriate physical inputs.

The common focus of the instructional system is the realisation of such human potential equipped with the needed educational inputs in all its variety and scope

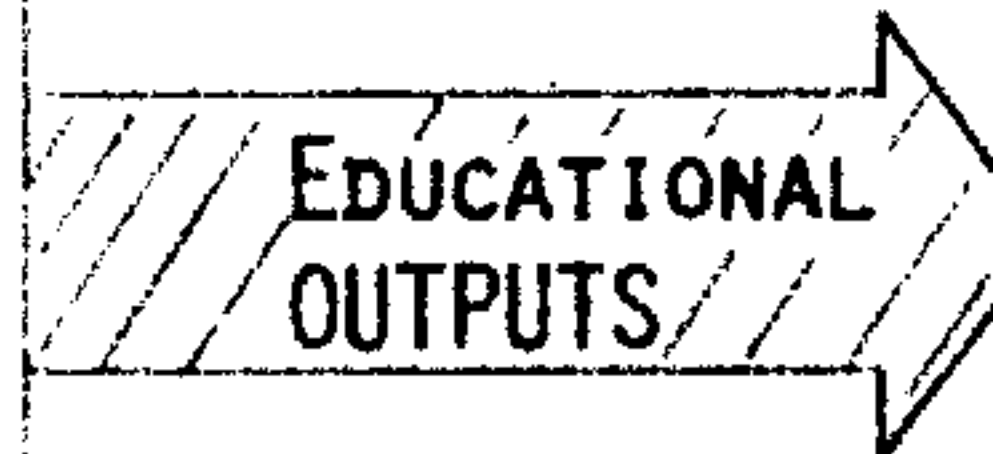
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THE MAJOR COMPONENTS OF AN EDUCATIONAL SYSTEM

— EDUCATIONAL PROCESS —>

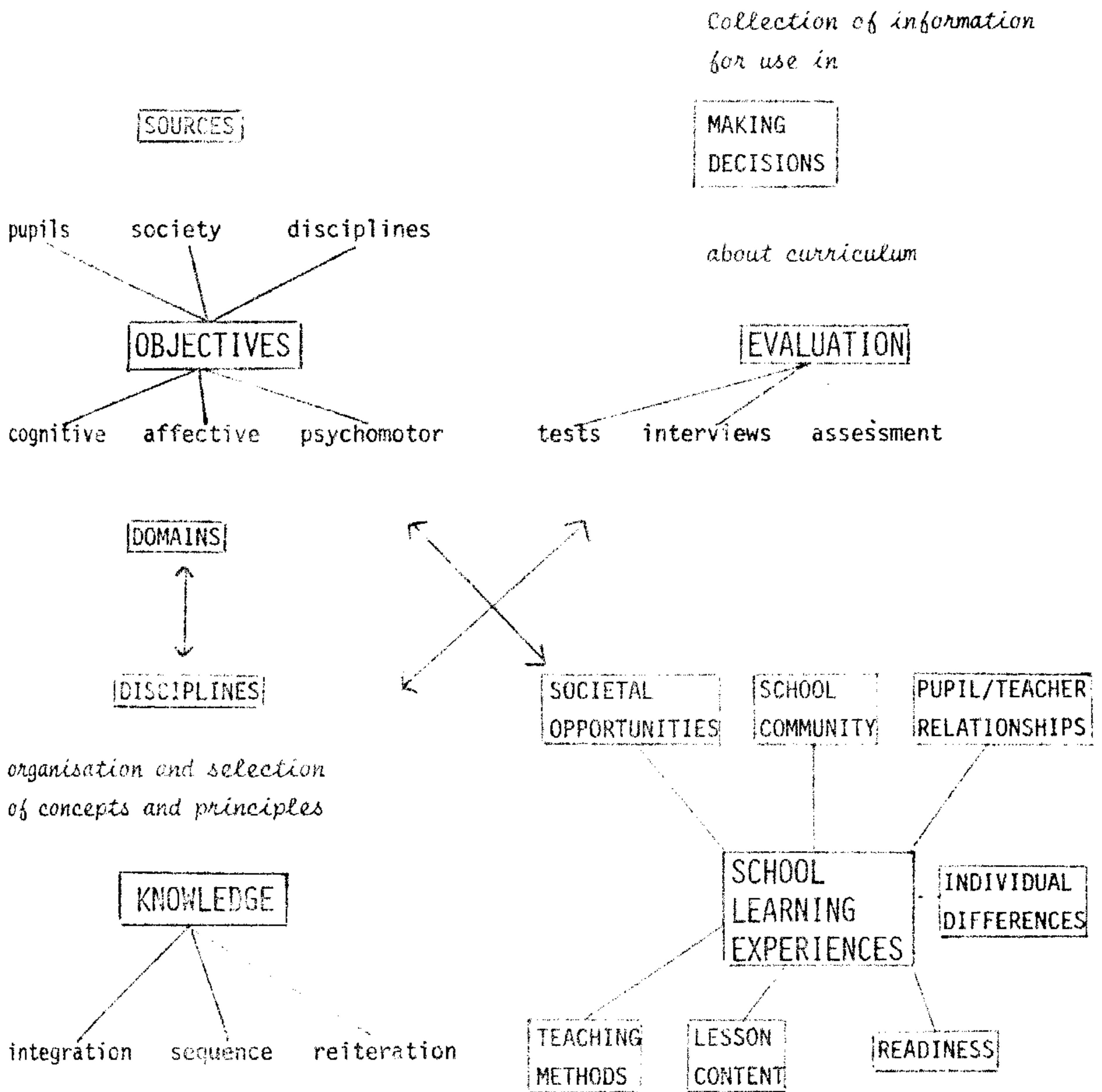


1. Aims and priorities
to guide the system's activities
2. Pupils
whose learning is the main aim of the system
3. Management
to coordinate, direct and evaluate the system
4. Structure and time schedule
to deploy time and student flows among different purposes
5. Content
the essence of what pupils are intended to learn
6. Teachers
to help provide the essence and guide the learning process
7. Learning aids
textbooks, blackboards, charts etc
8. Facilities
to house the process
9. Technology
all the techniques used in doing the system's work
10. Quality Controls
admission rules, examinations, standards
11. Research
to improve knowledge and the system's performance
12. Costs
indicators of efficiency of the system



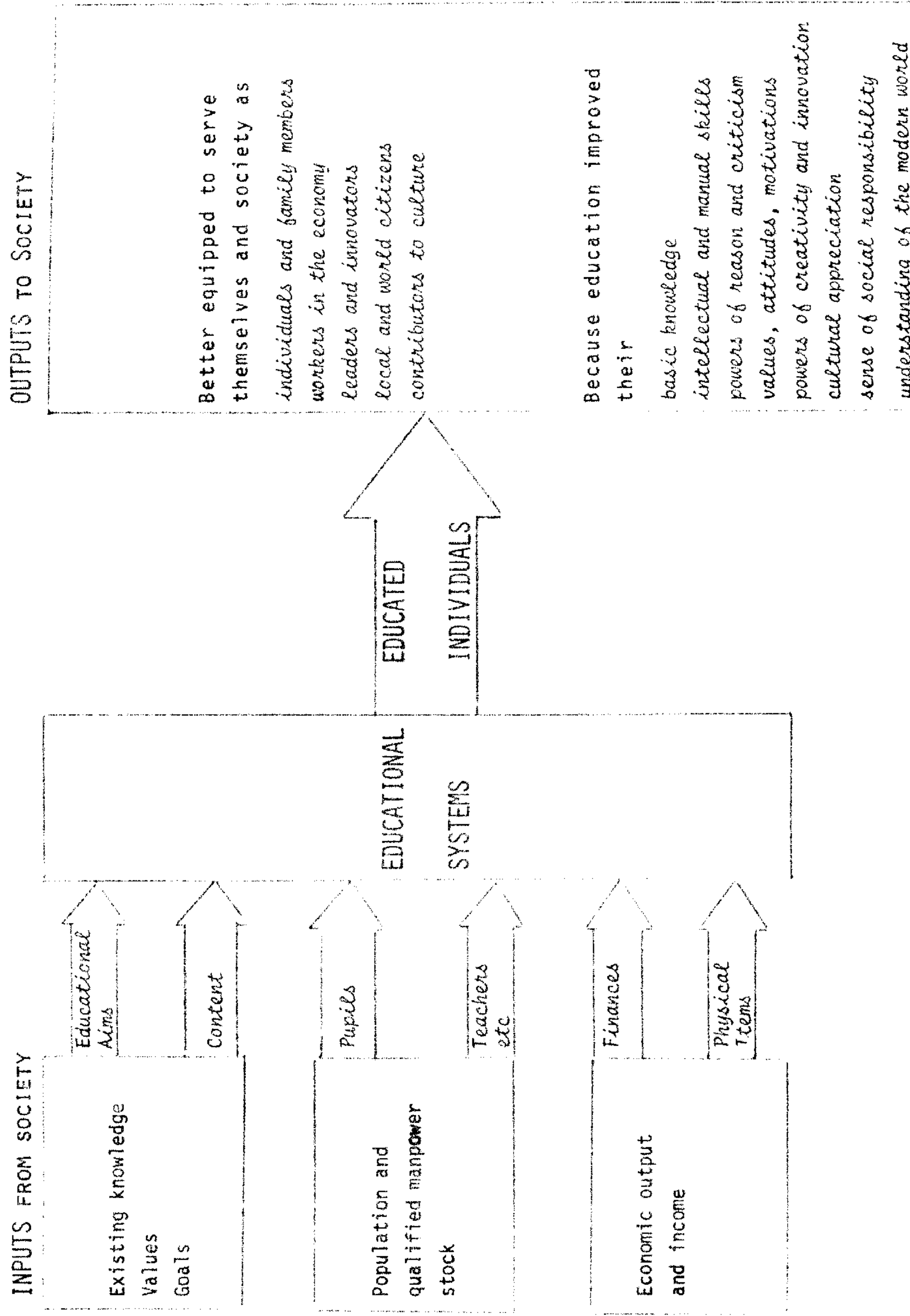
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INTERACTION BETWEEN AN EDUCATIONAL SYSTEM AND ITS ENVIRONMENT



A CONCEPTUAL MODEL OF THE INSTRUCTIONAL PROCESS

1. It is like a bed:

The four legs on which it stands are:

1. The Psychology of Learning
2. Testing and Evaluation
3. Curriculum Development
4. A Systems Approach

2. The four dimensions of this bed are:

1. Learning Materials
2. Teacher Competencies
3. An Instructional Model
4. Scientification of Operations

3. The warp/woof represents the kind of operations which go inside a classroom - the things that happen to kids.

4. The existing instructional process is highly ineffective and inefficient. It is the victim of a long state of inertia and stagnation. It is suffering from acute under-development.

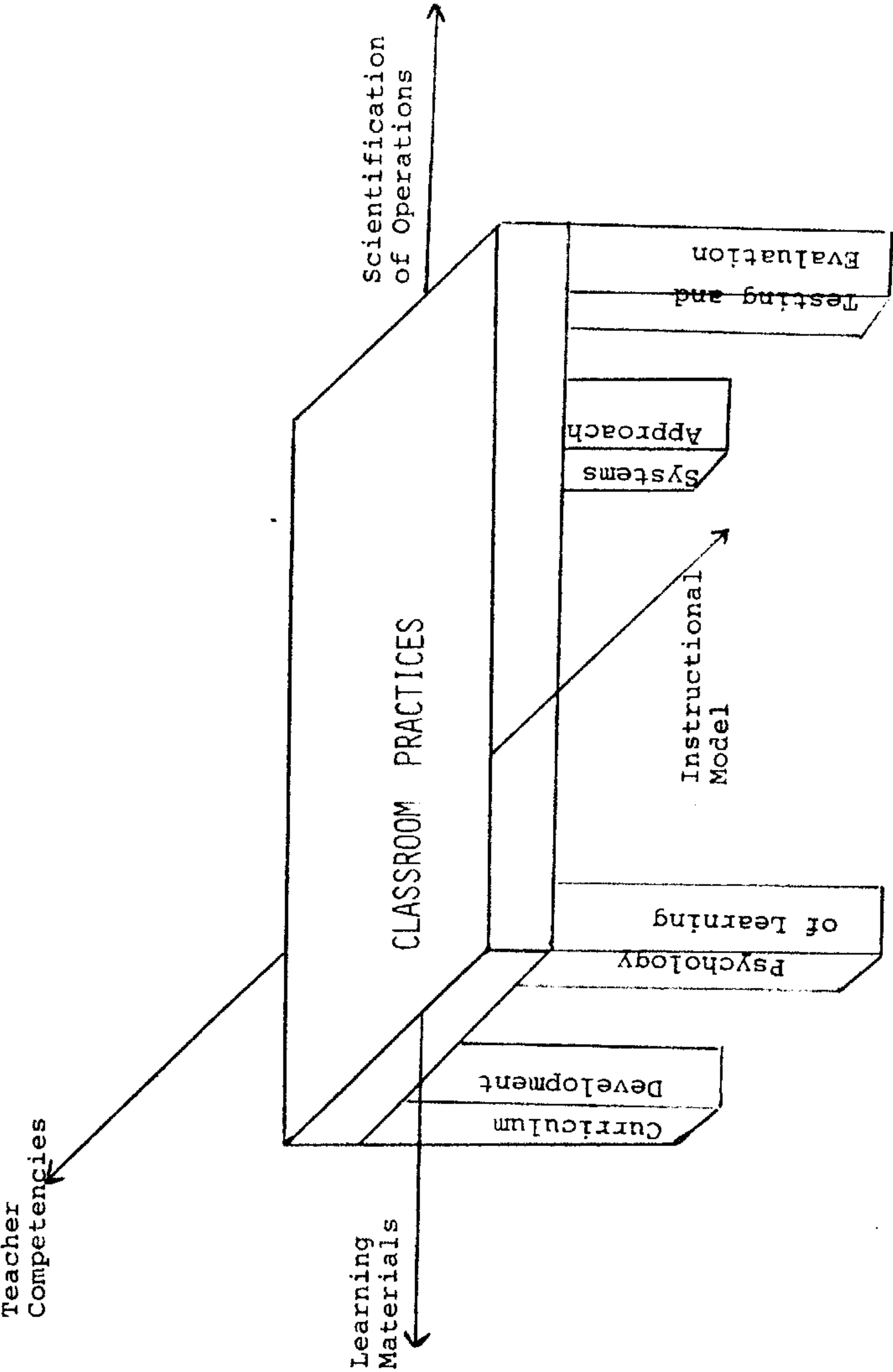
One or two parts of the bed are moth eaten and about to crumble - and some are missing altogether. The bed is in a highly unserviceable state.

5. In any effort aimed at instructional improvement, the level of awareness of the teaching force in service will need to be enhanced. Parts of the bed will have to be strengthened, replaced or provided.

6. The model is displayed visually overleaf.

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THE INSTRUCTIONAL PROCESS - A CONCEPTUAL FRAMEWORK





# EDUCATIONAL AIMS AND OBJECTIVES WITH SPECIAL REFERENCE TO PAKISTAN

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Strengthening Selected  
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in NWFP

## 1. AIMS, OBJECTIVES AND OUTCOMES

1.1 When we talk about the *aims of education* we usually mean pronouncements of vast scope and, often, vast vagueness. A typical example would be, "*to develop the personality of the child*".

1.2 Objectives are very SPECIFIC DESIRES and in Education they polarise into the objectives of the teacher and the objectives of the pupils.

A typical teacher's objective would be, "*This term I will teach my pupils to solve simple equations so that half of them will get half marks or more in the test at the end of the term*".

From the pupils' point of view the objective would be, "*By the end of this term, I will be able to solve simple equations at least half of the time*".

1.3 Over the past few years, a fair mystique has been built up about objectives, mainly because of curriculum building by a systems approach but there is little doubt of the importance from both the teachers' and pupils' point of view of knowing your objectives as specifically as possible.

As someone once said, *If you don't know where you are going, how will you know when you get there.*

Of course, a lot of the fun of Education is that you never know where you will end up when you start giving a course of instruction. Which brings us to outcomes.

1.4 Outcomes are what actually happen because of a classroom incident - or accident! Lots of outcomes are planned. Many more are unplanned. Lots of outcomes are desirable but we cannot pretend that all school activities lead to these. Another feature of outcomes is that many are not observed, certainly not by the teacher involved.

No-one can ever know the full results of an action they take. Stepping on a spider may result in an epidemic of flies in five years. Who knows? Some chance remark made to one child, may change another child's whole life.

- 1.5 When we state objectives we use OUTCOMES which are PLANNED and OBSERVABLE.

## 2. NATIONAL EDUCATION POLICY.

- 2.1 As mentioned in the first section, aims are wide desires. They may be stated quite precisely or they may only be hinted at.

Quite often aims only become really clear when the organisational structure of a department is designed. How are decisions reached? How are the children set or streamed in schools? How much money is spent in different areas? Which areas have plenty of staff? Which areas are always short of staff? What say have teachers, parents, pupils in the running of an institution? You can extend this list as you please and the reply given to each question will add to your knowledge of the aims of the institution you are studying. The written syllabus will also contain stated and unstated aims for you to unravel.

- 2.2 In addition to the *organisation* and the *syllabus*, there is a third area in which the aims of Education are seen in action. This is called the *interaction pattern*.

This refers to the attitudes of the people involved to each other. You might think of the relationships between:

*headteacher-teacher; teacher-teacher; teacher-pupil; parent-pupil; parent-parent.*

Their attitude to each other is a potent force in Education and gives an important guide to the educational aims. There is a tendency to ignore this when aims are under discussion but the whole topic of aims is beset with value judgements. What people make of them reflects their own beliefs and codes and so it is a subject that is difficult to write about in an objective way!

Keep in mind that aims are very often deeply hidden within a people and that the expressed aims are not always the most important and basic. It should be remembered also that the non-statement of these aims is rarely intentional. Usually they are so well understood and taken for granted that they are missed as being too obvious for comment.

- 2.3 Any educational policy ought to reflect the aspirations of the nation and embody principles of actions derived from philosophical choices considered most suitable for the achievement of the goals. When objectives change, Education must follow suit. It is for this reason that harmonization of Education in Pakistan with the concepts of Islam and the ideology of Pakistan necessitated the formulation of the present National Education Policy which was adopted in October, 1978.



- 2.4 Aims of Education are guide posts which provide purpose and direction to the educational system. Obviously they should be consistent with the people's faith, national ideology and aspirations. Since aims have to provide a clearcut framework, selection of them needs to be done boldly and with clarity of thought in order to provide a sound and meaningful base to the educational effort in Pakistan.

The Government has decided to adopt the following  
AIMS OF EDUCATION FOR THE NATION:

- 2.4.1 to foster in the hearts and minds of the people of Pakistan in general and the students in particular a deep and abiding loyalty to Islam and Pakistan and a living consciousness of their spiritual and ideological identity thereby strengthening unity of the outlook of the people of Pakistan on the basis of justice and fairplay.
- 2.4.2 to create awareness in every student that he, as a member of the Pakistani nation, is also a part of the universal Muslim Ummah and that it is expected of him to make a contribution towards the welfare of fellow Muslims inhabiting the globe on the one hand and to help spread the message of Islam throughout the world on the other.
- 2.4.3 to produce citizens who are fully conversant with the Pakistan movement, its ideological foundations, history and culture so that they feel proud of their heritage and display firm faith in the future of the country as an Islamic state.
- 2.4.4 to develop and inculcate in accordance with the Quran and Sunnah the character, conduct and motivation expected of a true Muslim.
- 2.4.5 to provide and ensure equal educational opportunities to all citizens of Pakistan and to provide minorities with adequate facilities for their cultural and religious development enabling them to participate effectively in the overall national effort.
- 2.4.6 to impart quality education and to develop fully, according to their capacity, each individual's potentialities through training and re-training and to develop the creative and innovative faculties of the people with a view to building their capability to effectively manage social, natural and productive forces consistent with the value system of Islam.
- 2.4.7 to provide a minimum acceptable level of functional literacy and fundamental education to all citizens of the country, particularly the young, irrespective of their faith, caste and creed in order to enable them to participate productively in the total national effort.

- 2.4.8 to create interest and love for learning and discipline among the youth and to ensure that every student is imbued with the realization that Education is a continuous and life-long process.
- 2.4.9 to promote and strengthen scientific, vocational and technological education, training and research in the country and to use this knowledge for socio-economic growth and development thereby ensuring a self-reliant and secure future for the nation.

2.5 While the debate about the relevance or otherwise of the educational goals has now subsided, there are some unresolved problems agitating the minds of those who have to concretize them in the field situation. There has always been a great deal of discussion about these problems but it has added a new urgency in the context of the current Islamic resurgence in the world. There is no easy solution to these problems as the history of Education in this sub-continent since the middle of last century has shown.

The problems are not simple. They have many dimensions and a multiplicity of complex issues both of manner and matter, form and content, techniques and purpose are involved.

A consideration of some of these issues in terms of the concrete experiences of the past reveals the necessity for a less dogmatic and more open-minded approach.

These points of conflict are very specific to the people of Pakistan and emerge from their geography and history, and as a result of the special identity that they bear to the world of today and yesterday.

Some of the issues are:

- 2.5.1 *The conflict between tradition and revolt.* This involves, for instance, the attitude towards some modern and even ultra modern concepts concerning teaching/learning strategies, up to classroom management patterns.
- 2.5.2 *The borrowing of techniques from foreign countries, especially the West.* A hundred years ago the then leaders of cultural development raised the slogan, "FOLLOW THE WEST". Today a powerful section, even those who have adopted the Western modes and diction, have adopted a "hate the West" attitude. This has taken the form essentially of opposition to Western social and cultural ideas but not Western techniques in Arts and Sciences. How is it possible to accept the techniques of measurement which are value free without accepting the concomitant moulds in which they are shrined?
- 2.5.3 *The history of ideas and the Sciences.* The philosophical tradition of the Muslims in the hayday of philosophical and scientific culture was one based squarely on ancient Greece. Much of the classical Muslim philosophy is derived from and comprises commentaries on the works of the Greek thinkers.

The cosmological system on which the philosophical and mystical ideas of Muslims is based is Ptolemaic, the Geometry is Euclidean, the numbers are Indian. The paradox is that these Egyptian, Greek and Indian ideas have been defended against the modern products of Western scientific developments as if they were part of Pakistan religious heritage and hence sacred.

- 2.6 These are only some of the problems with which thinkers have had to contend in the last one hundred years, and especially since the creation of Pakistan. As we have seen as a result of concrete experiences in the past and the present, there are no easy solutions to these problems.

They are all issues belonging to the realm of the mind and, like all issues of technique, form and content, cannot be solved in a simplistic or dogmatic form. Some persons try to be dogmatic and reduce every factor and mental function to a personal interpretation of what is Islamic and what is un-Islamic. The result is a total rejection of the entire history of human thought, including that part of it that Pakistanis have fondly regarded as their own for many centuries.

This extreme xenophobia, if it is adopted as the supreme criterion, can result in complete paucity of ideas, inhibiting all creativity.

- 2.7 There are two interconnected aims. They deal with the *development of the child* and the *development of the nation*. From these twin aims and the nature of the subject matter and of modern civilization there comes DERIVED AIMS and it is these that directly shape a syllabus.

## OBJECTIVES

- 3.1 Objectives are *specific desires* of the teacher and pupils and they are usually *outcomes which are both planned and observable*.
- 3.2 Objectives are given various names in the literature. You will see :  
behavioural objectives  
instructional objectives  
performance objectives.
- 3.3 A BEHAVIOURAL OBJECTIVE has three features.
- 3.3.1 There is the learner's PERFORMANCE which shows the result of his learning.
- 3.3.2 There is the LEVEL OF PERFORMANCE which shows how well the performance is carried out. (Aims might range from complete mastery to getting it right half of the time. Thinking of someone singing might help to distinguish the performance from the level of performance.)
- 3.3.3 The final feature of a behavioural objective concerns the CONDITIONS under which the performance is to be given.

- 3.4 An INSTRUCTIONAL OBJECTIVE is a behavioural objective in a teaching/learning situation.

An instructional objective might read like:

*By the end of today's lesson Ahmed will be able to solve a simple linear equation of the type  $x + 2 = 7$  getting at least 5 correct out of 10 in a 5-minute test.*

There is an INSTRUCTIONAL ELEMENT, a PERFORMANCE, a PERFORMANCE LEVEL and CONDITIONS for the performance.

- 3.5 A PERFORMANCE OBJECTIVE is a simplified type of instructional objective and is often called a modified instructional objective.

A performance objective only needs to say what performance is expected. Here is one:

*After the instructional period, Ahmed will be able to solve simple linear equations of the type  $x + 2 = 7$ .*

Most objectives are written in performance terms.

- 3.6 Objectives can be classified in many ways but the most common taxonomy used is Bloom's (q.v.). Let us look here at a modified version here.

There are three levels of mathematical thinking. The lowest level is RECALL or RECOGNITION where the learner repeats to the teacher exactly what was taught in the same form. The next level is reserved for ALGORITHMIC THINKING where the learner uses a well-drilled procedure to get a solution. The task involves picking the right procedure and carrying it out as taught. The vast majority of school mathematics is of this sort. The highest level of mathematical thinking is OPEN SEARCH. At this level no procedure already taught is sufficient on its own and the learner gets his own insight into the problem.

It is not a matter of age. All these levels are present in all teaching situations. In a University some facts are given to be memorised and will be asked at some point. In a primary school, flashes of insight occur regularly.

The three levels of thinking are associated with five of Bloom's levels. These are:

knowledge  
comprehension  
application  
analysis  
synthesis.

The top level, evaluation, is missing. It is thought that, for example, finding an error in a proof is not a higher skill than constructing the proof in the first place.



We have:

MATHEMATICAL THINKING  
LEVEL

Recognition, Recall  
Algorithmic Thinking  
Open Search

BLOOM'S OBJECTIVE  
LEVEL

Knowledge  
Comprehension, Application  
Analysis, Synthesis

KNOWLEDGE is shown by the ability to repeat what has been taught in exactly the same form. *What is  $16 \div 4$  ?* would test knowledge.

COMPREHENSION is shown by the ability to use algorithms mainly. It involves the knowledge of the algorithm involved but not the specific question asked. *What is  $3129 \div 23$  ?* test comprehension unless this specific division has been drilled. Also included in comprehension is the ability to give examples from definitions. *Give a prime number between 30 and 35* tests comprehension if the learner has knowledge of the definition of a prime number but has not memorised all the prime numbers up to 35.

APPLICATION usually involves changing from a verbal situation to symbols and then using one or two algorithms to complete the process. It is sometimes hard to draw a firm line between comprehension and application but the clue is in the unfamiliarity of the material. *45% of a group wear glasses. 22 do not wear glasses. How many are in the group?* might well be testing application.

ANALYSIS is shown by the solution of a problem which is entirely new to the learner and as the name suggests requires the learner to look at the relationships between the various parts of a problem. *Find a positive integer,  $t$ , such that  $t^2 - s^2$  is a prime number and  $s = 16$ .*

SYNTHESIS is shown by the ability to analyse the parts of a problem and use principles from another source in the solution. *In how many ways can the fraction  $\frac{1}{2}$  be written as a sum of two positive fractions with numerators 1 and denominator a natural number.*

The higher levels of analysis and synthesis are rarely shown in lists of performance objectives although they are highly desirable. Although we all want our pupils to operate at the open search level we need to concentrate on the basic foundations. It is up to the teacher to make a pupil aware of the power he holds. It may sound rather trite but each and every pupil must think about what he is doing. It is no good him doing 20 examples of a concept, identical except for the numerical values, unless he is conscious of what he is doing. A variety of material must be provided all the time if a pupil is not to go into a zombie-like unthinking state doing example after example as if he is in a trance.

The level of a particular objective depends on the previous teaching. A problem might tax a high school pupil at the open search level but a University student faced with the same problem may deal with it at the level of recall as he has met it several times before.

A question asked at the open search level in a National examination is quite likely to be drilled down to the lowest level by the following year.

A classification used by the International Evaluation of Achievement used slightly different categories of objectives:

1. Knowledge and information of definitions, notations and concepts.
2. Techniques and skills; computation; manipulation of symbols.
3. Comprehension; capacity to understand problems and to translate symbolic forms; to follow and extend reasoning.
4. Application; of appropriate concepts to unfamiliar mathematical situations.
5. Inventiveness; reasoning creatively in Mathematics.

Another simple classification which is useful is:

- A. Operations and techniques; straightforward manipulation or application of formulae.
- B. Problem solving requiring choice of operation.
- C. Understanding of concepts; problems requiring insight or ingenuity.

The words used in an objective give a good clue as to the level:

KNOWLEDGE: *choose, define, draw, identify, indicate, locate, select.*

COMPREHENSION: *calculate, compute, classify, convert, derive, describe, estimate, explain, express, factorise, list, solve.*

APPLICATION: *apply, assemble, demonstrate, distinguish, make.*

ANALYSIS: *analyse, deduce, determine, organise.*

SYNTHESIS: *design, develop, investigate, produce.*

This paper has concentrated on cognitive objectives but teachers, of course, have responsibilities in the domains referred to as affective and psychomotor as well. The affective domain is dealt with in detail elsewhere.

#### 4. OBJECTIVES - FRIENDS OR FOES

The advantages are:

Teachers know exactly what they are expected to teach.

Pupils know exactly what they have to learn.

Teachers know well in advance what is needed and so they can be well prepared.

Teachers can make plans which are realistic.

Teachers can ensure that appropriate learning activities are ready.

A pupil can evaluate his own progress.

The teacher knows exactly the achievement of each pupil.

A pupil is measured against himself, not his fellows.

Teachers can see when an objective is not being reached.

Progression is easily planned.

However :

The freedom of a teacher  
is limited.

The pupils have no freedom of choice.

The teachers become inflexible and miss out on unexpected teaching opportunities.

Much trivial work is done as trivia is easier to describe in an objective form.

The teaching is dull and dehumanised.

The pupil feels he is being judged all the time.

Each pupil feels the teacher is always on his back.

The teacher is made to feel  
he is accountable by results.

There is a very restricting system of regular tests.

Coherence is destroyed and conceptual structures are not developed.

No matter how the pros and cons of objectives are viewed, however, objectives remain the most convenient way for one teacher to tell another what it is he intends teaching.

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# THE OBJECTIVES OF PTC TEACHER TRAINING IN N.W.F.P.

DC/PAK/77/039/ED  
Strengthening Selected Educational  
Institutions in N.W.F.P.

## 1. INTRODUCTION

The objective of the PTC courses in Pakistan is to produce primary school teachers with the following *qualities*.

## 2. KNOWLEDGE

A primary teacher should have

- 2.1 a wide general education
- 2.2 an interest in learning things himself
- 2.3 a general cultural knowledge
- 2.4 an awareness of current affairs
- 2.5 specific knowledge of the ideological foundations, history and culture of Pakistan
- 2.6 complete knowledge of the content of the curriculum for classes I to V
- 2.7 mastery of the skills developed in classes I to V
- 2.8 a working knowledge of
  - 2.8.1 child development
  - 2.8.2 cognitive psychology
  - 2.8.3 counselling
  - 2.8.4 school organization
  - 2.8.5 classroom management
  - 2.8.6 principles of education
  - 2.8.7 methods of teaching
- 2.9 the ability to express his thoughts
  - 2.9.1 in speech
  - 2.9.2 in writing.

## 3. TEACHING ABILITIES

A primary teacher should be able to

- 3.1 speak clearly with a pleasant voice
- 3.2 explain matters clearly and simply
- 3.3 speak accurately
  - 3.3.1 the local language
  - 3.3.2 Urdu
- 3.4 avoid the use of
  - 3.4.1 harsh words
  - 3.4.2 slang
- 3.5 prepare lessons carefully
- 3.6 choose suitable learning episodes
- 3.7 understand the nature and needs of a child
- 3.8 analyse and make use of the pupils' environmental background



- 3.9 make use of the pupils' own experiences
- 3.10 select the best teaching strategy from
  - 3.10.1 play-way
  - 3.10.2 activity
  - 3.10.3 project
  - 3.10.4 unit
  - 3.10.5 problem solving
  - 3.10.6 question-answer
  - 3.10.7 drill
  - 3.10.8 role playing
  - 3.10.9 group discussion
  - 3.10.10 dramatization
  - 3.10.11 story telling
  - 3.10.12 field trip
- 3.11 make good use of class time
- 3.12 keep a balance between teacher work and pupil work
- 3.13 keep a good emotional atmosphere in the classroom
- 3.14 use a variety of methods and techniques
- 3.15 locate inexpensive teaching aids
- 3.16 use aids to enhance a lesson
- 3.17 prepare aids which are
  - 3.17.1 well-designed
  - 3.17.2 clear
  - 3.17.3 large enough for the purpose
  - 3.17.4 sufficient in number
- 3.18 use the following aids effectively
  - 3.18.1 chalkboard
  - 3.18.2 flannel board
  - 3.18.3 maps
  - 3.18.4 charts
  - 3.18.5 models
  - 3.18.6 pictures
  - 3.18.7 teaching kit materials
- 3.19 write clearly on
  - 3.19.1 chalkboard
  - 3.19.2 paper
  - 3.19.3 slate
  - 3.19.4 takhti
- 3.20 generate active discussions
- 3.21 present material in an interesting way
- 3.22 make topics meaningful for children
- 3.23 motivate children
- 3.24 deliver concept teaching lessons
- 3.25 encourage pupil participation
- 3.26 give pupil reinforcement
- 3.27 analyse individual differences in pupils
- 3.28 individualise the teaching
- 3.29 ask
  - 3.29.1 suitable formative questions
  - 3.29.2 probing and thought-provoking questions
- 3.30 vary the stimuli throughout a lesson
- 3.31 give clear and accurate directions
- 3.32 make good use of examples
- 3.33 induce a cognitive set quickly

- 3.34 close a lesson properly
- 3.35 give suitable assignments and homework
- 3.36 teach mixed ability classes
- 3.37 teach groups of children within a class
- 3.38 teach a class containing more than one grade
- 3.39 teach slow learners and provide them with remedial material
- 3.40 teach fast learners and provide them with enrichment material
- 3.41 complement the textbook
- 3.42 supplement the textbook.

#### 4. EVALUATION

A primary teacher should be able to

- 4.1 write
  - 4.1.1 attainment tests
  - 4.1.2 aptitude tests
  - 4.1.3 diagnostic tests
- 4.2 write test items such as
  - 4.2.1 essay
  - 4.2.2 structured response
  - 4.2.3 true/false
  - 4.2.4 multiple choice
- 4.3 identify the cognitive level of an item
- 4.4 calculate simple statistics
- 4.5 interpret simple statistics
- 4.6 analyse test results for diagnostic purposes
- 4.7 assess attitudes.

#### 5. PLANNING

A primary teacher should be able to

- 5.1 write objectives in performance terms
- 5.2 choose suitable objectives for a lesson
- 5.3 write clear and comprehensive lesson notes
- 5.4 plan a complete programme of work.

#### 6. CLASS CONTROL

A primary teacher should be able to

- 6.1 gain attention
- 6.2 hold attention
- 6.3 anticipate and avoid disciplinary problems
- 6.4 solve disciplinary problems
- 6.5 control unexpected situations
- 6.6 maintain routine activities.

#### 7. MANAGEMENT

A primary teacher should be able to

- 7.1 arrange the room in the best way to promote learning
- 7.2 make the best use of the available light
- 7.3 organise proper
  - 7.3.1 ventilation
  - 7.3.2 heating
  - 7.3.3 cooling

- 7.4 Keep the classroom
  - 7.4.1 neat
  - 7.4.2 clean
  - 7.4.3 attractive
- 7.5 keep the children prompt
- 7.6 instil in the children habits of
  - 7.6.1 neatness
  - 7.6.2 cleanliness
- 7.7 handle routine matters efficiently
- 7.8 prepare a well-balanced timetable
- 7.9 keep a neat and accurate record of
  - 7.9.1 attendance
  - 7.9.2 accounts
- 7.10 give necessary leadership
- 7.11 communicate effectively with
  - 7.11.1 the children
  - 7.11.2 colleagues
  - 7.11.3 superiors
  - 7.11.4 parents
  - 7.11.5 the community at large
- 7.12 supervise as required
- 7.13 convene and conduct meetings in school and out
- 7.14 act as a judge as required.

#### GUIDANCE

A primary teacher should

- 8.1 apply his knowledge of counselling to problems faced by
  - 8.1.1 the children
  - 8.1.2 their parents
  - 8.1.3 the community
- 8.2 run suitable co-curricular activities such as
  - 8.2.1 games and sports
  - 8.2.2 bazme-adab
  - 8.2.3 drama
  - 8.2.4 celebration of religious and national days
  - 8.2.5 school museum
  - 8.2.6 school garden
  - 8.2.7 contests for speeches
  - 8.2.8 Na'at Khani
  - 8.2.9 Husn-i-Qirat
  - 8.2.10 Cubbing
  - 8.2.11 Blue Bird,

#### PERSONAL MODEL

A primary teacher should

- 9.1 have a firm belief in his faith and not interfere with the faith of others
- 9.2 have a firm belief in the ideology of Pakistan
- 9.3 be a symbol of good character and conduct including
  - 9.3.1 austerity
  - 9.3.2 punctuality
  - 9.3.3 regularity

- 9.4 have a well developed sense of humour
- 9.5 be in his public dealings
  - 9.5.1 honest and forthright in discharging his duties
  - 9.5.2 actively involved in efforts to improve the educational, moral and spiritual life of the community
  - 9.5.3 willing to avoid associating with undesirable elements
- 9.6 be in his dealing with children
  - 9.6.1 just and impartial to all
  - 9.6.2 able to recognise individual differences among children and to encourage and guide them towards socially acceptable goals
  - 9.6.3 able to maintain individual records of the children and to keep their parents and related agencies informed about their progress
  - 9.6.4 seen to be constantly striving to enforce discipline among the pupils
  - 9.6.5 capable of understanding the pupils' problems and difficulties and of helping to solve them promptly
- 9.7 be in his dealings with his colleagues
  - 9.7.1 cooperative and friendly and capable of developing a relationship based on goodwill
  - 9.7.2 willing to treat other members of the profession in a way he would like to be treated himself
  - 9.7.3 careful to avoid discrediting any fellow teacher and doing anything that would discredit the teaching profession
  - 9.7.4 careful to avoid gossiping in class or out
- 9.8 be in his professional obligations
  - 9.8.1 a loyal and diligent worker discharging his duties honestly
  - 9.8.2 prepared to achieve the objectives laid before him in the accomplishment of his duties
  - 9.8.3 willing to take a personal interest in classroom and field activities
  - 9.8.4 willing to take part in orientation and retraining programmes
  - 9.8.5 able to make the teaching profession attractive
  - 9.8.6 prepared to avoid engaging in other gainful employment.

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SOME ASPECTS OF LEARNING

1. Instructional Skills

The skills that a teacher has can be referred to in total as 'teacher' skills. Within those skills is another set of skills, instructional skills or 'teaching' skills.

People in different areas of education require expertise in various areas. An officer dealing with examinations would need to be competent in evaluation, curriculum and management for example.

A class teacher has to be particularly skilled in instruction skills even though many other skills are needed to a lesser degree.

2. Theories of Learning

Think first about the brain. This is where learning takes place.

The brain looks after four different things:

- *It is responsible for the state of alertness of an individual. This is the reticular formation.
- *It monitors the organic needs of the various systems throughout the body. This is the hypothalamic activity.
- *It controls our emotions. This is the limbic system.
- *It carries out intellectual pursuits. This takes place in the cortex.

As teachers it is the cortex that most concerns us.

Before we leave the brain in general this is an important feature of it that is worth noting. The left-hand side of our brain is most concerned with thinking and the right-hand side with reflexes. Since messages from our left are dealt with first in the right-hand side of our brain, left-handed people seem to have an edge when it comes to sports like fencing and squash where instant reactions are important.

Let us return to the cortex. As we go from childhood to adulthood, more and more links are set up in the cortex enabling us to cope with increasingly complicated situations in the environment. The reason usually given for this slow development in the cortex is that too great an input of information into the brain of a child would overload the system and cause organic breakdown. It is true that many infant prodigies have short life spans.

This slow development in a normal child means that there are stages in the growth of a child's intellectual abilities and that you cannot expect a child to accomplish tasks at a higher level than the neural connections can cope with.

A very young child cannot be toilet trained until the appropriate nervous linkages have developed. A child will be unable to add until he is ready internally. Then he will learn to add or not depending on the teaching he is given. Some tasks will be beyond certain individuals permanently because they have a physical barrier in the brain that no amount of training will overcome.

The environment surrounding a growing organism will affect the rate at which the neural links are formed. The more complex the environment the more 'intelligent' the individual will become relative to a comparable individual in a non-stimulating atmosphere. This restraint can be much diminished by correct schooling.

Why does modern man seem so much smarter than his predecessors when their brains have apparently changed little over tens of thousands of years? The answer seems to be that the environment of ancient man, although difficult by our standards, was much simpler than ours. The intellectual stimulus was missing so the connecting links were not developed in the way ours are. Your children will grow up in a more complex society than you experienced and so they will be smarter than you, in general. How they will use this of course is something else again.

Every child will develop at a different rate and reach various important milestones at different times. Different cultures, or sub-cultures within a given culture, will produce children who are psychologically, intellectually and physically unique. The only thing that makes a national schooling system possible is that children are more alike than they are different. However, we must NEVER conceive of a body of knowledge that all children should acquire at a definite age in a definite sequence. Schooling has to deal with children as if they are all alike. Education within schooling looks at each child as an individual.

Education without schooling is only possible in very simple societies where the obligations and privileges of an adult are clear-cut and pre-ordained. It is hard to imagine any modern society without a formal school system. Some individuals whose role in the future of their society is certain may be able to opt out of the system. This lack of mobility exists only at the very top and very bottom of society. A poor farmer in a remote area may consider that his son need only learn the skills of a farmer and can do so best by helping him in the fields from an early age. Literacy and numeracy can come to the son through informal methods. A ruler can arrange for his son to be trained by a system of tutors or experts in various fields without requiring him to go near a school.

Before we leave the physical side of the learning process we should think how we interact with the environment. Information about our environment gets into our system through our receptors - our senses. The central nervous system carries this data to our effectors which take the most appropriate action in the light of our previous experience.

Modern learning theories revolve around the idea of conditioning but before we deal with this we must clarify what we mean by learning and behaviour.

Behaviour, in the psychological and educational sense, is not good or bad or acceptable or unacceptable. No value judgements are attached to it. Behaviour is what an individual does in response to a stimulus. The stimulus 'What is a bird?' can evoke a variety of behaviours. The one we wish to encourage will contain the characteristics of a bird. Other behaviours will be discouraged.

Learning is defined in terms of behaviour. Learning is a change in behaviour.

There are three implications from this:

1. Learning is an activity. To prove that learning has taken place we must demonstrate that there has been a change in behaviour.
2. Learning can take place without formal teaching. Teaching is a stimulus that has to compete with a host of other experiences for the attention of the potential learner.
3. Learning is a much wider activity than is normally considered. For example, a glutton has 'learned' to eat too much. A smoker has 'learned' to enjoy tobacco.

The actual learning which takes place is a more or less permanent change in the central nervous system. This cannot be seen. What we can see, however, is the performance associated with the learning. We hope that the external performance and the internal structure parallel each other.

Let us return to conditioning. This word again has no values attached to it as used in education. If conditioning is used for ulterior motives it is often referred to as 'brain-washing' but it is used here in its non-emotive sense.

To understand conditioning we need to consider reflexes. The natural response to a stimulus is an unconditioned response. That is the case when no training has been given. We hear a loud noise (the stimulus) and we jump (the response). This combination of stimulus/response is a reflex and as the response is natural in this case it is called an unconditioned reflex.

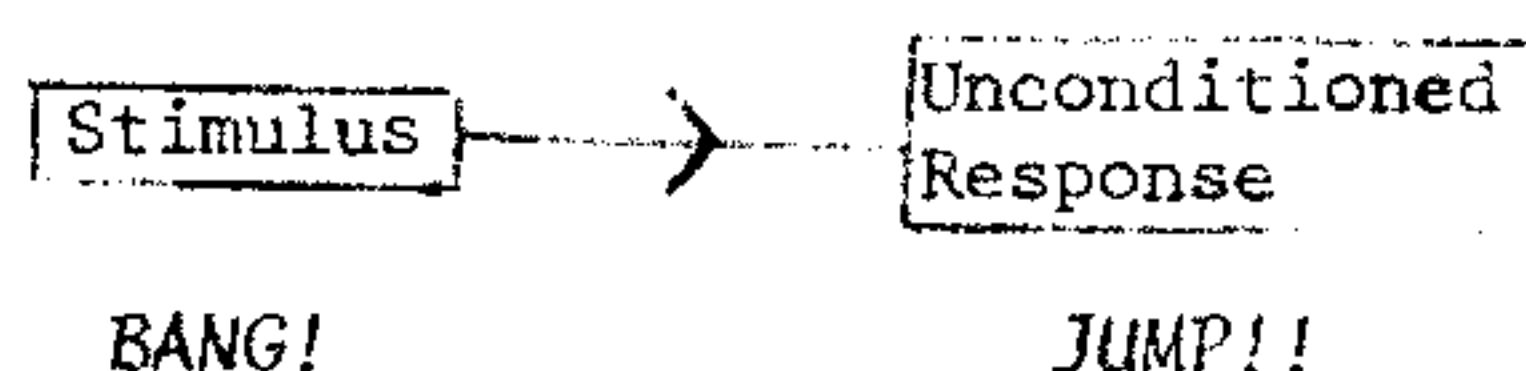


FIG.1 AN UNCONDITIONED REFLEX

Suppose now a soldier has been trained with weapons. His response to a sudden loud noise might well be to throw himself flat behind the nearest protective object. This response has been learned. We now have a stimulus and a conditioned response. This combination of stimulus/conditioned response is a conditioned reflex.

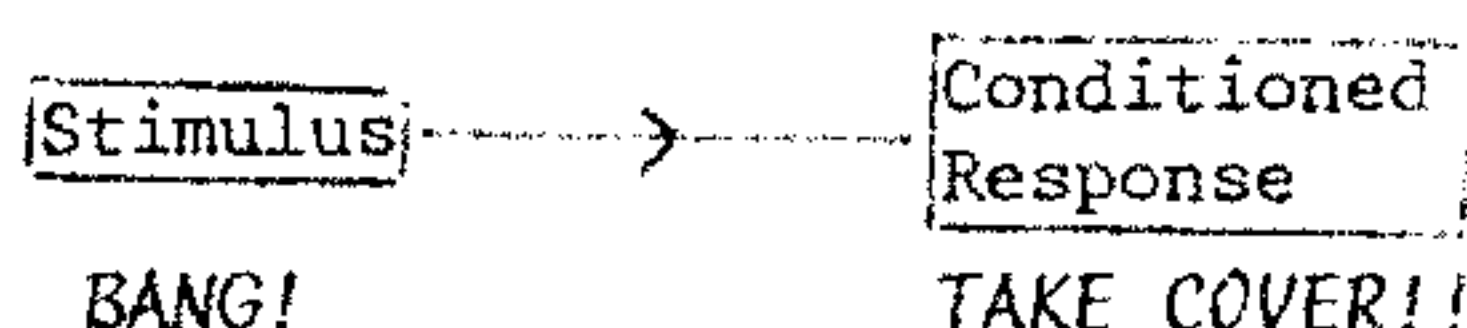


FIG.2 A CONDITIONED REFLEX

A conditioned reflex needs reinforcing or it tends to revert to the original unconditioned reflex. When the soldier returned to his peaceful village he would soon stop taking cover at every loud noise.

The best way of organising reinforcement in an educational setting is to set things up in such a way that the conditioned response carries its own reinforcement. The wanted response should have desirable characteristics for the subject. This type of conditioning is called instrumental conditioning.

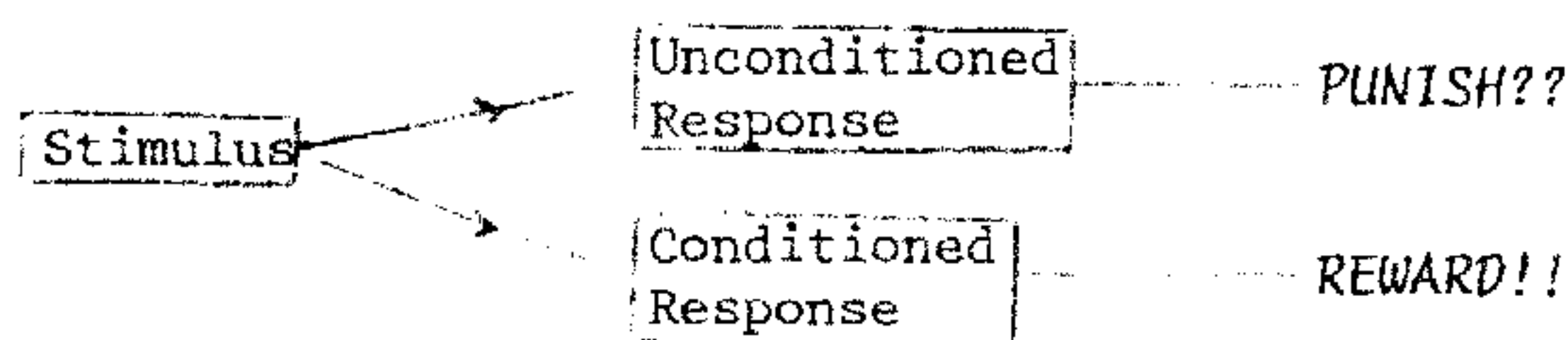


FIG.3 INSTRUMENTAL CONDITIONING

This stimulus/response view of learning leads to a hierarchy of learning skills:

1. Signal learning. A question leads to a single correct response. The stimulus, "How many grams in 3 kilograms?" leads to the response "3000".
2. Chain learning. A question leads to a sequence of responses before the correct response is reached.
3. Multiple discrimination learning. A question requires the learner to decide which of two or more chains has to be followed to get the correct response.
4. Concept learning. The learner can distinguish between one idea and other very similar ideas.
5. Principle learning. The learner can chain together groups of concepts into a large body of knowledge.

There is another view of learning which is not so mechanical as the stimulus/response theory outlined above. This is based on Gestalt psychology.

In the stimulus/response theory we learn by adding together all the elements we pick up through our senses. The holistic theory tells us that in fact the whole is greater than the sum of its parts. We do not only collect all the elements, we learn the relationships between them and it is how the organism integrates the various elements that decides how well we learn. A good analogy is a well-written novel. It is not the words or the sentences or the paragraphs or the chapters that make it a great novel it is how all these parts are fitted together. So it is with learning according to the Gestalt school.

Genuine learning takes place in bouts of sudden inspiration. An instant insight is gained into the problem and a complete solution can be seen without the need for step by step analysis. This is most clearly seen in Mathematics where one goes from being able to do only a limited set of examples to being able to cope with a vast area of the subject in a few moments. It can be seen in language where a foreign language suddenly falls into place after perhaps years of learning words and phrases.

In order to promote insights, the learner must be shown the widest possible picture at all times. Keeping his nose to individual leaves is a sure way to make sure that he never sees the forest.

Large areas of knowledge can be mastered in a relatively short time by those pupils who gain insight. When a new topic is introduced, everyone should be given the chance to make use of their intuitive feelings. The pupils should be told what it is you are about to do and shown the type of examples they will be able to do at the end of the piece of work. You may be surprised at how many will grasp the ideas without having to slog through the grind of graded examples you have prepared for them. Such an obstacle course is more likely to turn them off than tune them in.

Of course, most teaching depends for its bread and butter on the stimulus/response theory with a hope that insights will be formed quite often.

3. Concept Formation

It has been said that it is the human beings' grasp of concepts that makes their thinking different from all other living creatures. Our ability to link these concepts into large principles makes us the best adapted of all animals.

Concepts are tied up with language ability but what exactly is a concept. Think about a chair. You should have no difficulty with that but you might have difficulty in defining a chair in such a way that it would satisfy everyone and also cover every type of chair that has ever been made. When you sit on a desk or a wall do they become chairs? Anyhow you have the concept of a chair if you can classify any given object as a chair - or not.

Do you know what a 'flower' is? If you do you would have no difficulty in classifying a canna lily as a flower. You have the concept of a flower. You may have more difficulty with the concept of the lily. Could you be sure to classify every flower brought to you as a lily or a non-lily? If not, you do not have the concept of a lily.

4. Assimilation and Accommodation

When an individual reacts with his environment he might find it familiar or strange.

Assimilation refers to an individual looking at a specific object or event and being able to classify it in terms of the concepts he has already formed. You might think, "Ah, this is the same as..." or "This is very like..."

Accommodation refers to being faced with an object or event that will not fit in with your existing concepts. You may have to drastically alter a concept to cope with the new idea. You will think, "That's funny, I thought ..." or "See, I'll need to ...".

With assimilation the concepts you already have are reinforced. With accommodation the concepts you already have are found wanting and need to be altered.

A child thinks he knows what a cube is. His teacher gives him a die. He thinks, "That's right. Its the right shape. Its a cube". He has assimilated a die into his concept of a cube. The teacher now shows him a square prism. The pupil is a bit confused. "Its got 6 sides. It looks like a cube from this end - but not from the top. Its not a cube. Its something else." The pupil has extended his concept of the cube by a distracting non-example. This is accommodation.

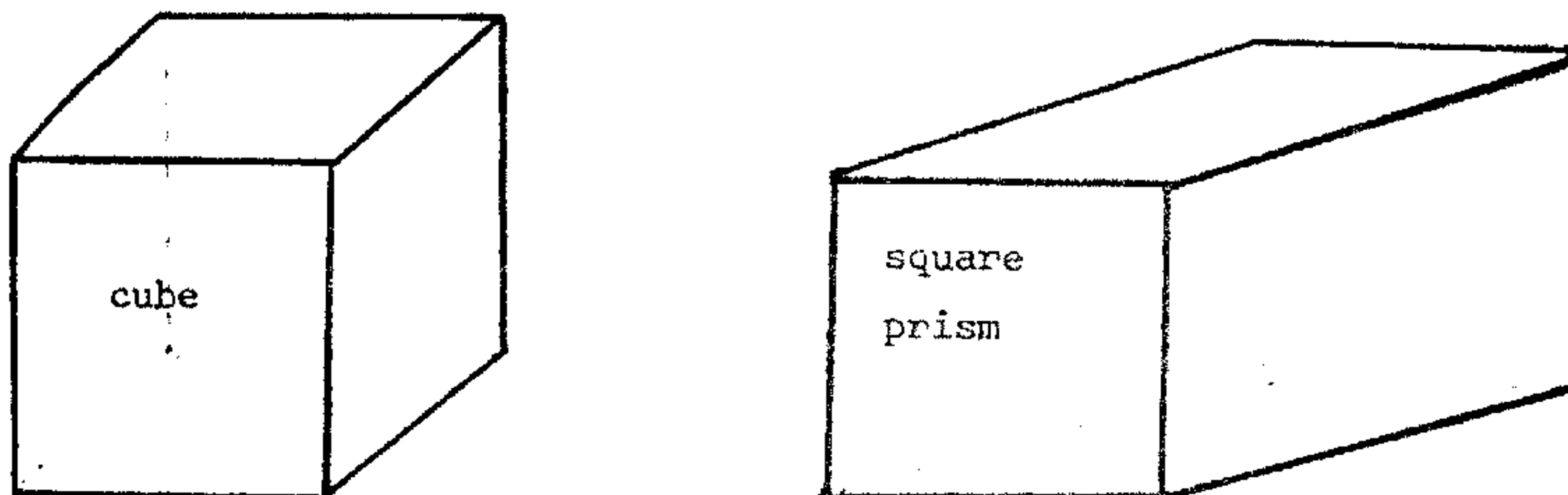


FIG 4. CUBES AND SQUARE PRISMS

5. Piaget's Stages

A child's ability to form concepts is not constant throughout his life. It alters with age and Piaget distinguished the following levels.

1. The sensory-motor stage. Up to 2 years of age the child's knowledge depends on concrete items. He can classify permanent objects.
2. Pre-conceptual intelligence. From 2 to 4 years, the child makes concepts but has difficulty with classes. All animals may be 'doggies' and all men 'daddy'.
3. Intuitive thinking. From 4 to 7 years, the child can form concepts but bases the classification on superficial appearance. The same amount of water will be 'more' when put into a thinner glass.

4. Concrete operations. From 8 to 11 years, the child can form complex concepts but they depend on concrete examples for their reality.
5. Formal operations. Over 12, a child can now reason without having to go back to concrete examples.

The ages given are only for guidance and also there is no sharp division between one level and the next.

It is at level 3 that the teacher is most likely to be fooled into thinking that a child knows more than he actually does. To such a child the world is a very plastic place where things constantly change their appearance. And it is precisely for this level that we are training teachers.

As well as being affected by age, concept formation can be affected by three things.

1. The type of teaching may not stimulate concept formation.
2. The culture will stress some concepts at the expense of others.
3. The language may assist or hinder the growth of a concept.

To illustrate this last point. There may be no word, or no specific word, for the concept that is required. If the word for a wing and a feather is the same, a concept that requires distinguishing between them will be hard to establish.

6. Some Definitions.

Teachers have to "think about thinking", "learn about learning", "form a concept of a concept" and so on. Let us sort out a few concepts.

KNOWLEDGE is of two types. SPECIFIC KNOWLEDGE is the information that we have about a certain topic. GENERAL KNOWLEDGE is our ability to handle specific knowledge. It can be called INTELLIGENCE.

Intelligence in action is THINKING.

We can THINK without learning but we cannot LEARN without thinking.

LEARNING is a change in BEHAVIOUR.

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THE MATHEMATICS LESSON

DC/PAK/77/039/ED

Strengthening Selected Educational : 1 1

Institutions in N.W.F.P.

1. The most important thing to realise is that Mathematics is learned by DOING MATHEMATICS. It cannot be learned by listening to a teacher, no matter how brilliant the teacher may be. Question and answer methods have very little place in the teaching of primary school Mathematics. It must be thought of as a PRACTICAL SUBJECT. Time spent with the pupils not working is time wasted.
2. A typical mathematics lesson should begin with a short, very short, exposition by the teacher, reminding the pupils about what they were doing in the previous mathematics lesson and telling them what they will be doing next. Only in exceptional circumstances will this take even $\frac{1}{4}$ of the time of the lesson. The mark of a good mathematics teacher is HOW QUICKLY HE CAN GET THE PUPILS WORKING ON THEIR OWN.
3. Learning Mathematics is not a communal activity though, of course, cooperation between pupils can be of help. It is very much an INDIVIDUAL PERSONAL subject and each pupil ought to be able to develop his potential in it to the fullest possible extent.
4. Mathematics is most easily learned when the material has meaning for the learner as DRILL MUST FOLLOW UNDERSTANDING where possible. However, sometimes it is only after a lot of drill that meaning becomes clear so a teacher should get as many pupils to understand as possible, then do the drill and graded examples, all the while attending to individuals to make them understand what they are doing.

5. Much of the time in a good mathematics lesson will be spent with the teacher talking to small groups of pupils or individual pupils while the rest of the class GET ON WITH THEIR WORK. A teacher should not feel, or be made to feel, that his place is at the front of the class talking to the whole group. The latter teaching method is not a suitable style and must be eliminated or at least reduced to a minor strategy.
6. The theory of learning most suited to Primary Mathematics is a mixture of STIMULUS-RESPONSE and GESTALT theories. It is critical in mathematics teaching to INDUCE INSIGHTS in the pupils. This requires two tactics:

6.1 The pupils must always be faced with the largest possible picture of what they are trying to learn. Too often the concentration on detail stifles the real learning that should be taking place.

6.2 The pupils must be made to see the relationships that exist between the items being learned. It is these that give substance to the holistic view that 'THE WHOLE IS GREATER THAN THE SUM OF THE PARTS'.

A well taught pupil can expect regular sudden flashes of understanding which will carry him well forward in his knowledge of Mathematics and can expect to master a topic completely without having to wade through every step in the process. A whole principle becomes clear at once.


Of course, the daily teaching is straight-forward stimulus-response under the teacher's guidance. BUT this should be accompanied by the teacher's influence in keeping the learner aware of what he is learning.

7. Mathematics consists of SKILLS and CONCEPTS. In order to master a skill over-learning is needed. That is, excessive drill must be given. This is particularly

important with number skills that are needed in everyday life. These are skills that everyone must be able to bring into use at any moment.

Concepts are developed through the use of language and the usual steps are:

- 7.1 A child can give examples of the concept and describe some of its properties
 - 7.2 A child can distinguish examples of the concept from examples of very similar concepts
 - 7.3 A child can define the concept by giving a list of its essential properties.
8. It must be clearly understood that concept formation in a primary school is rather different from concept formation in secondary schools and colleges.

In the early primary years, a child will be at the Piaget level of INTUITIVE THINKING. The essence of this is that the child forms quite clear concepts but they are based on superficial appearances. It is at this level that a teacher can be easily fooled into thinking that a child knows far more than he in fact does. A child who is taught that  is a cube may not recognise a real cube when he sees one. What he has seen is a flat figure with a square and two parallelograms.

A child at this level can hold quite conflicting ideas in his head without worry. In different situations the same question can have different answers. In one class he might learn that '1 is a prime number' and in another '1 is not a prime number'. He will be quite happy to give each teacher the reply he wants. To the child the world is a plastic place and reality varies from time to time and place to place.

The other major problem with concept formation at this level is that the child has not got the concept of conservation. There will be more water if it is poured

into a narrow glass from a fat one. The concept of SIZE will be particularly tricky. Does size mean height or width or weight or what? Is a man bigger than a donkey?

The older primary children will be at the level of CONCRETE OPERATIONS. Quite complex concepts can be formed but they depend on concrete examples or examples from the child's own experience. Genuine abstract thinking cannot be expected anywhere in a primary school.

Although concept formation is seriously affected by age there are three other factors which will affect it.

- 9.1 The type of teaching may encourage or restrict concept formation.
- 9.2 The culture of the North West Frontier Province will stress some concepts over another.
- 9.3 The language of the child and the teacher will assist or hinder the growth of a concept and this is of particular importance where several languages are competing in the child's mind for dominance at any one time.

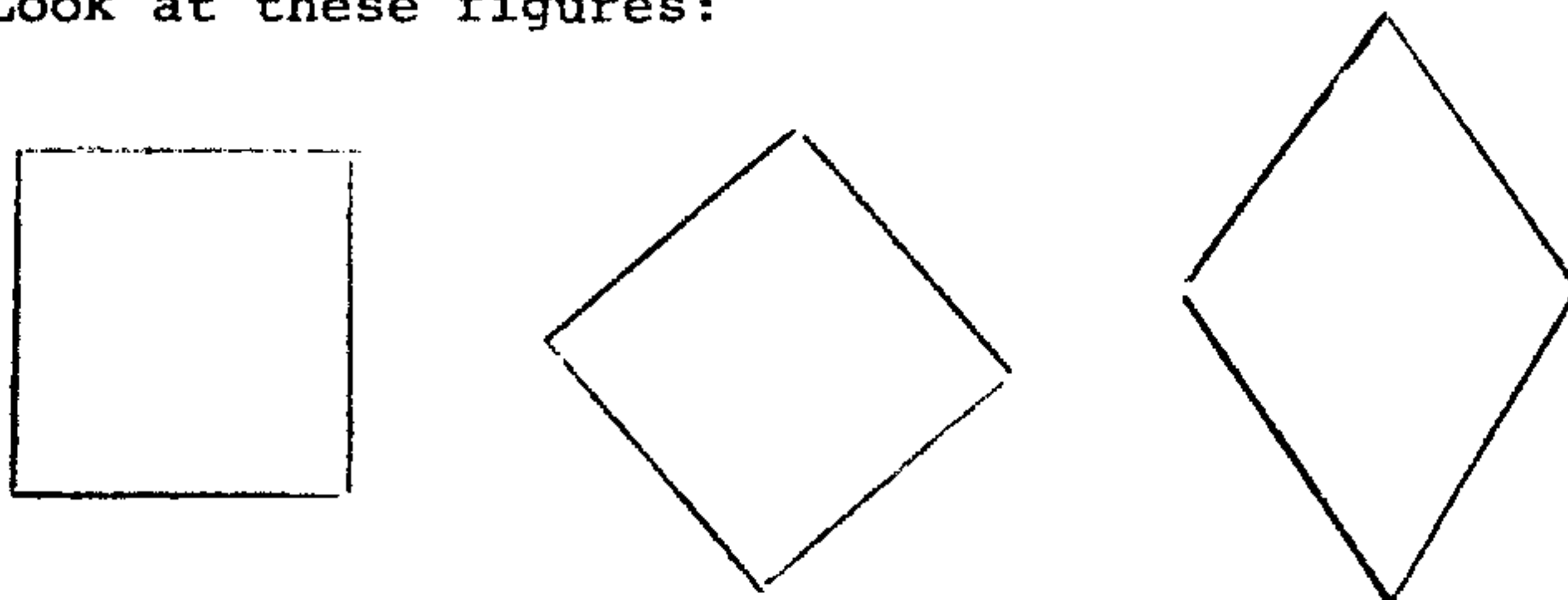
Teachers must keep firmly in their minds the difference between THE AIDS to teach a concept and THE CONCEPT itself. One example, is a number ray to teach the concept of addition and subtraction. If the concept to be taught is addition or subtraction the use of the ray should be discontinued as soon as possible.

If the lesson is about number sets or number rays the use of the number ray should be continued. The teacher must be absolutely clear just which concept he is trying to get across to the pupils.

Again a teacher should distinguish carefully the skill of doing certain additions and subtractions and the concepts of addition and subtraction themselves.

The main thing to get out of MODERN MATHEMATICS is accuracy in the use of language. No word should be used unless its meaning is perfectly clear to child and teacher alike. CONSISTENCY is all important. The correct word should be used for a concept from the start.

Look at these figures:



The first two are squares and the other is a rhombus.

To do the classification properly, the child needs to have the concept of a square and a rhombus really well fixed in his mind.

A young primary school child will be happy to call the first figure a square and the other two 'diamonds' or some such term. He must, however, be shown that it is the first two that are alike - not the second two.

12. You will see sometimes that with modern mathematics you should be both RIGOROUS and CONCRETE. This is not possible at the primary school level.

One obvious example is the empty set. Pupils make sets of objects on their desks then take items away until the set is empty. Do they all have empty sets? They cannot because there is only one empty set, THE empty set. It is not possible to have AN empty set. If I have the empty set no one else in the Universe can have it,

Another example is that all the number sets we are interested in are INFINITE sets and they cannot be demonstrated in a concrete way.

The moral is that a concept will be slowly refined throughout the primary schooling of a child and any attempt to rush the process will fail. The concept taught must be 'correct' at a particular maturity level but it must be capable of being evolved to the genuinely true concept at a later stage without having to be unlearned.

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## TYPES OF LEARNING ASSOCIATED WITH PARTICULAR A.V. AIDS

| TYPE OF AID    | LEARNING STRUCTURES |       |                |         |           |
|----------------|---------------------|-------|----------------|---------|-----------|
|                | Signal              | Chain | Discrimination | Concept | Principle |
| RADIO          |                     |       |                |         |           |
| RECORD PLAYER  |                     |       |                |         |           |
| TAPE RECORDER  |                     |       |                |         |           |
| LINE DRAWING   |                     |       |                |         |           |
| STILL PICTURES |                     |       |                |         |           |
| TRANSPARENCIES |                     |       |                |         |           |
| FILM           |                     |       |                |         |           |
| TELEVISION     |                     |       |                |         |           |

The hatched areas show which learning structure can be taught by each aid.

# THE COGNITIVE DOMAIN

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1. The cognitive domain is concerned with our intellectual activities and the level at which we are using them.
2. Bloom\* and his associates constructed a taxonomy for the Cognitive Domain which had six levels:
  1. Knowledge
  2. Comprehension
  3. Application
  4. Analysis
  5. Synthesis
  6. Evaluation.
3. At the level of KNOWLEDGE, the pupil can bring to mind the material with little or no alteration from the way it was presented to him. He remembers what he was told.
4. COMPREHENSION requires the pupil to have understood the material to the extent of being able to use it in simple situations.
5. The pupil's understanding is at the level of APPLICATION if he can use the material in particular and concrete situations.
6. At the level of ANALYSIS a pupil can uncover the relationships that exist between ideas that he is presented with.
7. When the pupil reaches SYNTHESIS he can put together elements in a problem in such a way as to create a structure that was not clearly there before.

Taxonomy of Educational Objectives - Cognitive Domain. David McKay, 1956

8. The final level is EVALUATION. Here the pupil must exercise his judgement based on criteria either prepared by him or given to him from another source.
9. Each of these six levels has sub-divisions<sup>\*</sup>:
  1. Knowledge
    - 1.1 Knowledge of Specifics
      - 1.11 Knowledge of Terminology  
e.g. is familiar with a large number of words as commonly used.
      - 1.12 Knowledge of Specific Facts  
e.g. knows the parts of a plant.
    - 1.2 Knowledge of Ways and Means of Dealing with Specifics
      - 1.21 Knowledge of Conventions  
e.g. knows the structure of a particular type of poetry.
      - 1.22 Knowledge of Trends and Sequences  
e.g. can trace the history of Pakistan culture.
      - 1.23 Knowledge of Classifications and Patterns.  
e.g. is familiar with a range of types of poems.
      - 1.24 Knowledge of Criteria  
e.g. knows the criteria for evaluating an essay.
      - 1.25 Knowledge of Methodology  
e.g. can list the steps in preparing a survey in social studies.
    - 1.3 Knowledge of the Universals and Abstractions in a Field
      - 1.31 Knowledge of Principles and Generalizations  
e.g. can recall the major generalizations about Pakistani culture.
      - 1.32 Knowledge of Theories and Structures  
e.g. can recall the properties of a group in Mathematics.
  2. Comprehension
    - 2.1 Translation  
e.g. can write a word problem in algebraic terms.

\* except 'Application'.

## 2.2 Interpretation

e.g. can grasp the meaning of a poem.

## 2.3 Extrapolation

e.g. is skillful at predicting trends.

### 3. Application

e.g. can apply the theory of algebraic groups to Geometry.

#### 4. Analysis

#### 4.1 Analysis of Elements

e.g. can distinguish facts from hypotheses.

## 4.2 Analysis of Relationships

e.g. can understand the interrelationships among the ideas  
in a poem.

### 4.3 Analysis of Organizational Principles

e.g. can recognise the techniques used in television advertising.

## 5. Synthesis

## 5.1 Production of a Unique Communication

e.g. can tell a personal experience effectively.

## 5.2 Production of a Plan, or Proposed Set of Operations

e.g. is able to produce a learning package.

### 5.3 Derivation of a Set of Abstract Relations

e.g. can make scientific generalizations from data.

## 6. Evaluation

## 6.1 Judgements in Terms of Internal Evidence

e.g. can pick out fallacies in arguments.

## 6.2 Judgements in Terms of External Criteria

e.g. can compare a work with the highest known standards  
in its field.

## EXERCISE ON COGNITIVE OBJECTIVES

Classify each of these according to Bloom's Taxonomy. Each one fits into a different category.

1. A can describe  $A = b \times h$  in words
2. B knows how to check hypotheses against given information
3. C can identify an example as belonging to Algebra
4. D is proficient in writing units for College courses
5. E knows the rationale of modern mathematics
6. F can define a rhombus
7. G can compare the culture of Pakistan with that of Saudi Arabia
8. H can list the essential elements of an epic poem
9. I knows what will happen if he adds a particular item to the diet of his sheep
10. J knows how to do science experiments
11. K has the ability to assess the accuracy of facts given in a report
12. L is able to list the functions of the roots of a plant
13. M is able to list various theories for the cause of the French Revolution
14. N is capable of making major scientific discoveries
15. O is good at writing short stories
16. P can identify unstated assumptions
17. Q can draw conclusions from memos written by his boss
18. R can explain why he felt fear during the showing of a film
19. S knows what to listen for during a poetry reading competition
20. T can trace the influence of science on everyday life
21. U is able to summarise the chapter of a book

# KEY TO THE EXERCISE ON THE COGNITIVE DOMAIN

|    |      |     |      |     |      |
|----|------|-----|------|-----|------|
| 1. | 2.1  | 8.  | 1.21 | 15. | 5.1  |
| 2. | 4.2  | 9.  | 3.1  | 16. | 4.1  |
| 3. | 1.23 | 10. | 1.25 | 17. | 2.3  |
| 4. | 5.2  | 11. | 6.1  | 18. | 4.3  |
| 5. | 1.31 | 12. | 1.12 | 19. | 1.24 |
| 6. | 1.11 | 13. | 1.32 | 20. | 1.22 |
| 7. | 6.2  | 14. | 5.3  | 21. | 2.2  |



## THE AFFECTIVE DOMAIN

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1. The cognitive domain. This is concerned with our intellectual activities and the level at which we are using them. The affective domain refers to the attitudes that we have to the concepts and principles that we are learning.
2. Bloom\*and his associates constructed a taxonomy for the Affective Domain but they found it to be quite a more difficult task than that of the Cognitive Domain. However, they finally arrived at the following five levels:
  1. Receiving
  2. Responding
  3. Valuing
  4. Organisation
  5. Characterisation.
3. At the level of RECEIVING, a pupil is willing to pay attention to what the teacher is presenting. This attention will need to be captured by the teacher at the lowest level but at the highest level the pupil will be making some sort of effort to concentrate on the lesson.
4. At the next level of RESPONDING, the pupil is not just a passive receiver but is sufficiently motivated to show an interest in the topic. The pupil will do the work of the class with an interest ranging from very low to quite intense. Left to himself, however, he would probably stop the activity.
5. VALUING is reached when a pupil undertakes a piece of work for its own sake, and not for the sake of the teacher. This is when a pupil realises the worth of some activity for him and carries it out consistently. The commitment can vary, at the lowest level, from casual to a strong desire, at the highest level, to convert others to his point of view.
6. ORGANISATION refers to a situation where more than one value has to be considered and the pupil has to establish some system of values. He will need to classify his values, deciding which are more important than others.
7. At the top level of CHARACTERISATION, the pupil has organised his values into a system which controls his behaviour in a rather automatic way. It is an integral part of him and he does not question it unless he is threatened in some way.

\*Taxonomy of Educational Objectives- Affective Domain by Krathwohl, Bloom and Masia. David McKay Co. 1964.

8. All of these levels have sub-divisions:

1. Receiving.

1.1 Awareness.

e.g develops awareness of fashion in dress

1.2 Willingness to Receive.

e.g. listens carefully when others speak

1.3 Controlled Attention.

e.g. listens to poetry with some discrimination

2. Responding.

2.1 Acquiescence to Respond.

e.g. obeys the school rules

2.2 Willingness to Respond.

e.g. reads about current affairs in the newspapers

2.3 Satisfaction in Response.

e.g. enjoys reading books of history

3. Valuing.

3.1 Acceptance of a Value.

e.g. has a growing desire to write effectively

3.2 Preference for a Value.

e.g. examines a variety of view points on controversial matters

3.3 Commitment.

e.g. has faith in the power of reason and discussion

4. Organisation.

4.1 Conceptualisation of a Value.

e.g. tries to identify the characteristics of his favourite form of poetry

4.2 Organisation of a Value System.

e.g. weighs alternative teaching methods against the benefits to the pupils rather than the teachers

5. Characterisation.

5.1 Generalised Set.

e.g. is ready to change judgements in the light of later evidence

5.2 Characterisation

e.g. has a consistent philosophy of life

9. Let us write down a possible thought in the mind of a person who has each of these attitudes:

| LEVEL | POSSIBLE THOUGHT                    |
|-------|-------------------------------------|
| 1.1   | Teacher is writing on the board.    |
| 1.2   | I'd better read the words.          |
| 1.3   | So that's what I am supposed to do. |

| LEVEL | POSSIBLE THOUGHT                                                                 |
|-------|----------------------------------------------------------------------------------|
| 2.1   | I'd better get this done or I will get punished again.                           |
| 2.2   | I know that. I will put up my hand.                                              |
| 2.3   | I know that, sir. Please let me do it, sir.                                      |
| 3.1   | I'll keep on working at my maths. I rather like it.                              |
| 3.2   | I must get Teacher to give me extra work. I really like this.                    |
| 3.3   | I will try to get a Maths Club going to get others to share my interest.         |
| 4.1   | I prefer Maths to Physics. It's more fundamental.                                |
| 4.2   | If I want to be a good mathematician I will need to study Physics as well.       |
| 5.1   | Now that I am a trained mathematician, how can I use this to help my fellow men? |
| 5.2   | I am happy to be a maths teacher.                                                |

Attitudes to many things are developed very early in life and are very difficult to alter thereafter. It has been said that if parents and teachers are consistent, a child's attitude to life will be pretty well established by the time he is eight.

If this is so a heavy burden falls on parents and primary school teachers to establish good habits. Other teachers must consolidate this into an acceptable adult life style.

## EXERCISE ONE ON AFFECTIVE OBJECTIVES

Each of the following behaviours fits into a different one of Bloom's affective taxonomy. Try to do the classification.

1. A listens for the rhythm in poetry read aloud.
2. B practices the rules of good health.
3. C has a sense of responsibility for participating in public discussion.
4. D observes with increasing differentiation the sights and sounds encountered in school and out.
5. E develops a conscience.
6. F relates his own ethical standards and personal goals through the reading of biography.
7. G changes his mind when new facts demonstrate the need for the revision of perviously held opinions.
8. H writes letters to the press on issues he feels strongly about.
9. I derives satisfaction in reading poetry with others.
10. J develops a tolerance for a variety of types of music.
11. K has a realistic attitude of an emotional adjustment to the limitations inherent in his own aptitudes, abilities, interests and physical conditions.
12. L is loyal to the groups of which he is a member.
13. M is willing to force himself to participate with others.

## EXERCISE TWO ON AFFECTIVE OBJECTIVES

Each of the following behaviours fits into a different one of Bloom's affective taxonomy. Try to do the classification.

1. N forms judgements as to the responsibility of society for conserving human and natural resources.
2. O views problems in objective, realistic and tolerant terms.
3. P desires to obtain optimum health.
4. Q visits museums when told to do so.
5. R listens to others with respect.
6. S responds with consistent, active and deep interest to intellectual stimuli.
7. T has faith in the power of reasoned argument.
8. U develops for the regulation of his own personal and civic life a code of ethical behaviour.
9. V is sensitive to the importance of keeping informed on current political and social matters.
10. W enjoys a constantly increasing variety of programs on radio.
11. X assumes responsibility for drawing reticent members of a group into conversation.
12. Y recognises that there may be more than one acceptable point of view.
13. Z develops techniques for controlling aggression in culturally acceptable patterns.

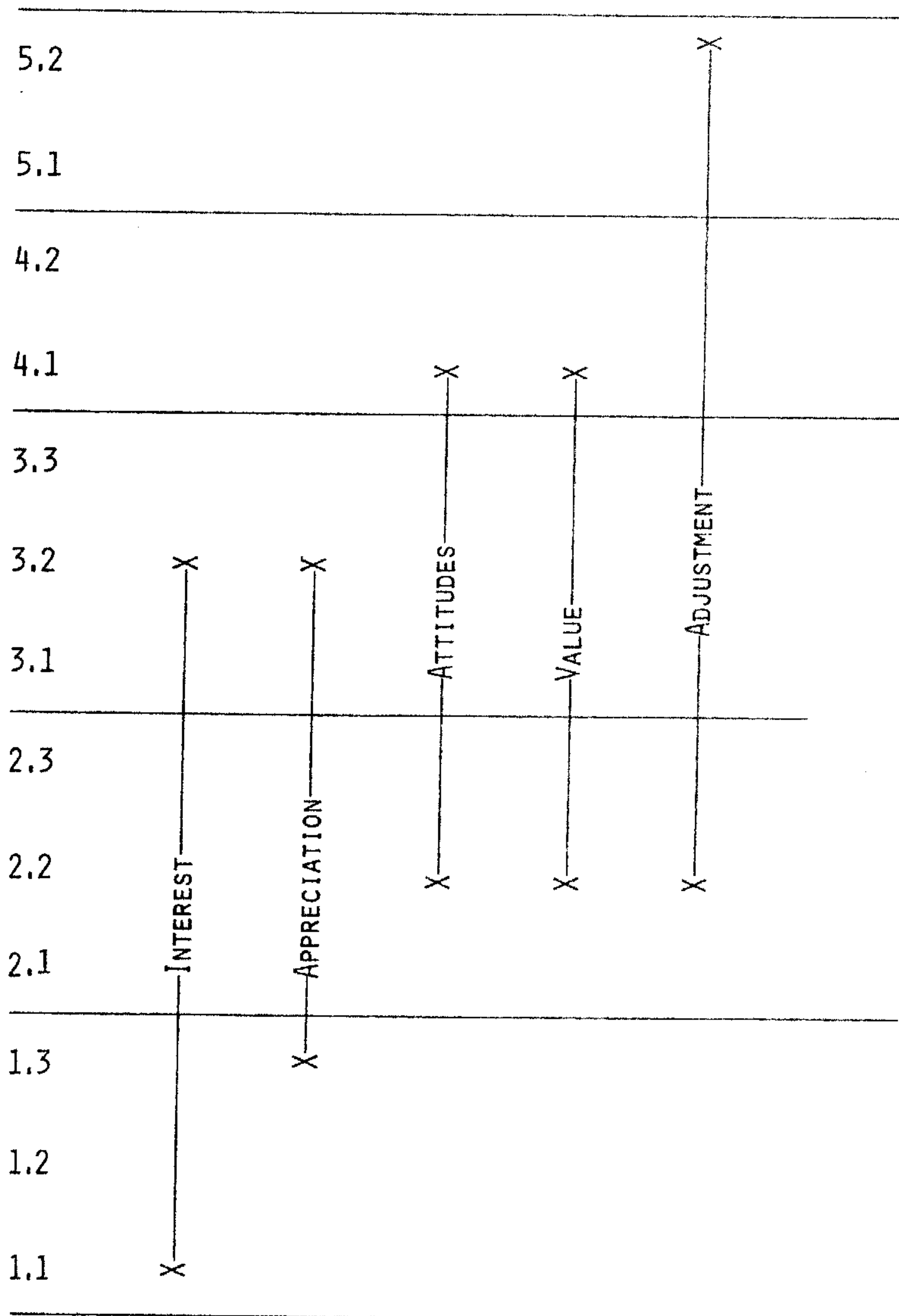


KEY TO THE EXERCISES ON THE AFFECTIVE DOMAIN TAXONOMY

The following are the classifications given by Bloom.

|               |     |    |     |    |     |    |     |    |     |
|---------------|-----|----|-----|----|-----|----|-----|----|-----|
| Exercise One: |     |    |     |    |     |    |     |    |     |
| A.            | 1.3 | B. | 2.2 | C. | 3.1 | D. | 1.1 | E. | 5.2 |
| F.            | 4.1 | G. | 5.1 | H. | 3.2 | I. | 2.3 | J. | 1.2 |
| K.            | 4.2 | L. | 3.3 | M. | 2.1 |    |     |    |     |
| Exercise Two: |     |    |     |    |     |    |     |    |     |
| N.            | 4.1 | O. | 5.1 | P. | 3.1 | Q. | 2.1 | R. | 1.2 |
| S.            | 2.2 | T. | 3.3 | U. | 5.2 | V. | 1.3 | W. | 2.3 |
| X.            | 3.2 | Y. | 1.1 | Z. | 4.2 |    |     |    |     |

# RELATIONSHIP OF CERTAIN PERSONALITY TRAITS WITH AFFECTIVE TAXONOMIC LEVELS



# THE COMPARISON OF SUBJECT-CENTRED AND LEARNER-CENTRED CLASSROOMS

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| AREA            | SUBJECT-CENTRED/TEACHER-DOMINATED<br>CLASSROOM                                                                                                                                                                                                                                                                                                                  | LEARNER-CENTRED<br>CLASSROOM                                                                                                                                                                                                                                                                                                                                                                  |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1. ORGANISATION | <p>The pupils are assigned to a group for the whole session.</p> <p>Each pupil has a fixed schedule.</p> <p>A pupil is assigned to a fixed classroom for the session.</p> <p>The pupil is in a graded classroom from which there will be promotion at the end of the session.</p> <p>The pupil is in a self-contained classroom presided over by one adult.</p> | <p>The pupils are grouped according to the activity.</p> <p>A pupil will have a flexible schedule which will vary according to need.</p> <p>A pupil will work in small, medium and large groups in various work places.</p> <p>The pupil is in a non-graded classroom with continuous progress opportunities.</p> <p>The pupil is in a team-taught school and reacts with several adults.</p> |
| 2. KNOWLEDGE    | <p>Pupils concentrate on facts.</p> <p>The knowledge comes within disciplines.</p> <p>The knowledge is convergent.</p> <p>The knowledge is sequenced by subject needs.</p> <p>The knowledge is mostly low level cognitive.</p> <p>The knowledge comes from one or two textbooks.</p>                                                                            | <p>Pupils learn concepts and processes.</p> <p>The knowledge is inter-disciplinary.</p> <p>The knowledge is both convergent and divergent.</p> <p>The knowledge is sequenced by the needs of the pupil and the subject.</p> <p>The knowledge is cognitive, affective and psychomotor.</p> <p>The knowledge comes from various primary and secondary sources.</p>                              |

### 3. METHODOLOGY

The teacher asks questions to which there is only one correct response which the teacher knows. (didactic)

The teacher only recognises one or two learning styles.

Mass coverage is emphasised - from cover to cover.

The teacher interacts with the pupils.

The teacher emphasises the textbook.

### 4. EVALUATION

Evaluation is periodic, at the end of a session.

The evaluation is carried out by the teacher.

Closed questions are used.

Tests are norm-referenced.

Most tests use paper and pencil.

The questions come from the subject area.

Evaluation is restricted to the cognitive domain.

The evaluation is of the pupil for grading.

The didactic method is balanced by an enquiry approach.

The teacher uses several modes of teaching and accepts various styles of learning.

There is an in-depth approach to fewer topics.

As well as teacher-pupil interaction, the pupils interact with one another and the material.

Multi-media material is used.

Evaluation is continuous.

The evaluation is done by the teacher and the pupil.

Open questions are often asked.

Most tests are criterion-referenced.

As well as paper and pencil tests, checklists, rating scales, observations, profiles and conferences are used.

Questions are based on concepts.

Evaluation covers, cognitive, affective and psychomotor domains.

Evaluation is for planning and tests the effectiveness of the material.

# A COMPARISON OF THE TYPES OF COURSES REFERRED TO IN N.W.F.P., AS THE SEMESTER SYSTEM AND THE ANNUAL SYSTEM.

DC/PAK/77/039/ED

Strengthening Selected Educational  
Institutions in N.W.F.P.

## 1. INTRODUCTION

There has been considerable controversy in Pakistan over the introduction of the type of education course called a *semester system*. This has been compared both favourably and unfavourably with the contrasting type of course called an *annual system*. This paper attempts to clarify the characteristics of each system and to list the arguments for and against each system.

## 2. THE CHARACTERISTICS OF THE ANNUAL SYSTEM

- 2.1 A course runs for a complete academic year. In a few cases it runs for two years.
- 2.2 A composite examination is held at the end of the course.
- 2.3 This final examination carries great weight.
- 2.4 Both internal and external examinations are given.
- 2.5 Internal examinations carry little weight.
- 2.6 The material is presented in the form of lectures and a few set textbooks.
- 2.7 Only in practical subjects is work given of a practical nature.
- 2.8 A rigid body of knowledge has to be mastered by a student in a fixed time.

## 3. THE CHARACTERISTICS OF THE SEMESTER SYSTEM

- 3.1 A course is divided into many parts or units.
- 3.2 Assessment is made of each unit as it is finished.
- 3.3 There is no external examination.



- 3.4 There is no composite final internal examination.
- 3.5 The material is presented in a variety of forms in addition to lectures and textbooks. Wide use is made of personal assignments.
- 3.6 A great deal of practical activity is generated in every course.
- 3.7 There is flexibility in the order in which the students can tackle a unit and in the length of time they can spend on a unit.

#### SUBJECT-CENTRED AND LEARNER-CENTRED INSTRUCTION.

Much of the argument has at its heart the desirability or otherwise of one of the above positions. Should the instruction be subject-centred or learner-centred?

To avoid undue repetition, you are referred to the paper on *The Comparison of Subject-Centred and Learner-Centred Classrooms* which forms part of this series.

#### THE MERITS OF AN ANNUAL SYSTEM

The structure of an annual system is said to have the following merits:

- 5.1 Only a few textbooks are needed to cover a course so the cost is small.
- 5.2 Examinations are based directly on the work in the textbooks so the teachers' and students' work can be sharply focussed.
- 5.3 A teacher can spread the work out evenly over the duration of the course without a low workload at one time and over-pressure at another.
- 5.4 Teachers and students can enjoy long vacations without the burden of preparation or assignments.
- 5.5 An efficient student with intelligence and a good memory is rewarded with success.

- 5.6 There is no need to attend the course regularly so a student can attend to other matters at the same time as studying.
  - 5.7 A teacher has only one major examination to prepare for and the student has only one important examination to sit.
  - 5.8 Good writing skills carry great weight with the examiner.
  - 5.9 High marks can be obtained in the examination.
  - 5.10 A course can be easily changed.
  - 5.11 If a teacher is replaced, there is little effect on the course.
- The teacher gets the following advantages:
- 5.12 The teacher has freedom to teach as he wishes.
  - 5.13 The teacher is not held accountable for the performance of the students.
  - 5.14 The teacher cannot be accused of partiality towards particular students as he is not the final judge.
  - 5.15 The work of a teacher is light.
  - 5.16 The teacher can use the same material with successive classes of students.
  - 5.17 A teacher has sufficient spare time to devote to other matters.
  - 5.18 Discipline can be enforced in a strict formal way.
  - 5.19 The teacher can be a dignified aloof person interested only in his subject and unaffected by the personal problems of the students.
  - 5.20 The teacher can ensure that the course is covered and that standards are preserved.
  - 5.21 The institution has the academic appearance that the general public expects with attentive students sitting in ranks listening to the wisdom of an eminent lecturer.
  - 5.22 The institution has the well disciplined look appreciated by the public and officialdom

5.23 The teachers have status.

5.24 A teacher has the opportunity to be a private tutor as his material is retained by him as lecture notes and not handed out as handouts.

*Note: An Annual System can be run on the cheap if it is restricted to lectures supplemented by a single textbook and little or no provision is made for a good library, alternative texts, mimeographed material, suitable buildings, study rooms, practical work and field trips.*

## 6. THE MERITS OF A SEMESTER SYSTEM

The structure of a semester system is said to have the following merits:

6.1 The material is presented in small manageable units.

6.2 Students' attendance is essential otherwise they will miss important work and assessments.

6.3 Both teachers and students have to work hard throughout the year.

6.4 Teachers and students have to cooperate.

6.5 Mutual confidence is built up between teacher and student.

6.6 Fair dealing is required from both teacher and student.

6.7 A teacher gets to know the students well.

6.8 Perseverance and hard work are as important as intelligence and memory.

6.9 A teacher can give a highly accurate assessment of every student because of the close relationship and continuous assessment.

6.10 Students develop good learning habits.

6.11 Students' attitudes are improved.

6.12 There is an informal friendly working atmosphere.

6.13 Cramming is of little use.

- 6.14 No-one can pass a course by luck.
- 6.15 Creative work is possible.
- 6.16 The practical work develops in the students a good attitude to using their hands.
- 6.17 The students become research oriented.
- 6.18 The knowledge gained is personalised and so stands a good chance of being used.
- 6.19 Good social habits are formed by working in cooperation instead of competition.
- 6.20 The students quickly get to know if they are coping with the work of the class and if not have a chance to remedy it at an early stage.
- 6.21 Students who complete a course are more or less assured of passing.

The teachers get the following advantages:

- 6.22 The teacher is not under pressure to cover the course as his time is used in helping the learners.
- 6.23 The teacher has a lot of modern aids.
- 6.24 The teacher has the major say in the grading of his students.
- 6.25 The teacher can use a wide variety of teaching skills other than lecturing.
- 6.26 The teacher gets plentiful and continuous feedback to enable him to improve the course.
- 6.27 The teacher learns from the students about his subject and about their ideas and philosophies.
- 6.28 There is a reduction in the chance of alienation between the teacher and the students.
- 6.29 The teacher gets the credit or blame for his students' performances.

*Note: A Semester System by its nature cannot be run on the cheap. To try to do so destroys it.*

*One advantage of introducing a semester system is that new materials and equipment are needed as well as new ideas. The users of these can then be HELD ACCOUNTABLE FOR THE SUCCESS OF THE SYSTEM.*

*The capital costs of modern curricula can often be found from sympathetic agencies who are in agreement with the need if they can see good use being made of previous inputs.*

## 7. THE DISADVANTAGES OF THE ANNUAL SYSTEM

The annual system is said to have the following disadvantages:

- 7.1 Only a limited number of textbooks is used.
- 7.2 The students loaf through most of the course and then overwork in the last few weeks with consequent anxiety and tension.
- 7.3 The external examination does not assess accurately what has been covered in a course. Even at best, a final composite test can only be a sample of the course work. It cannot cover all aspects of the course.
- 7.4 Stress is laid on those items most likely to appear in the final examinations and so single unrelated topics are taught rather than principles.
- 7.5 Too much of the year is wasted on holidays.
- 7.6 It is hard to maintain discipline as students do not need to attend classes on a regular basis.
- 7.7 Teachers tend to be alienated from their students because of their contrasting roles.
- 7.8 Intelligence and memory are of more importance to a student than hard work and perseverance.
- 7.9 Guides and primers are used instead of the proper subject development found in a textbook.
- 7.10 Few assignments are given and the material is given to the students by lectures and dictation.



- 7.11 A student can get through a course with a little knowledge and a lot of luck.
- 7.12 The teacher covers the course irrespective of the students' aptitudes, interests or well-being.
- 7.13 The main learning technique is cramming.
- 7.14 A dishonest student has opportunities for passing by unfair means.
- 7.15 A student can fail a whole year because of a minor problem arising during the final examinations.
- 7.16 No credit is given for creativity. In fact a creative answer stands a good chance of being harshly penalised compared to the streotype answer which is expected.
- 7.17 No practical activities are carried out unless there is absolutely no option.
- 7.18 The students are discouraged from having an enquiring nature. They are receptors of knowledge.
- 7.19 The purpose of a course is to get as many students as possible through the examinations, not to give them an education.
- 7.20 Competition is stressed instead of cooperation.
- 7.21 The students do not think of the teacher as a person but as a giver of information.
- 7.22 There is a long delay after the completion of a course before a student knows his results.
- 7.23 A lot of expenditure goes into a little learning.
- 7.24 Inconsistency in marking can affect a student's whole life without him being able to check its accuracy.
- 7.25 A student does not know until after the course is completed whether he is coping or not.
- 7.26 A teacher becomes lazy and repeats the same course exactly year after year.

*It is the sort of education of which it has been said that 'the knowledge passes from the notebook of the teacher to the notebook of the student without passing through the mind of either'!!*

## 8. THE DISADVANTAGES OF THE SEMESTER SYSTEM

The semester system is said to have the following disadvantages:

- 8.1 There is a heavy workload on both teachers and students over the whole year and, at times, this builds up to impossible levels.
- 8.2 The teachers need a great deal of knowledge and skill before they can successfully implement the system.
- 8.3 Much space and material has to be provided.
- 8.4 It is difficult for a student to get high marks in open ended situations.
- 8.5 There are too many tests.
- 8.6 A student who is absent from part of a course finds it hard to catch up.
- 8.7 The limited resources available make the implementation impossible.
- 8.8 A change in teacher can seriously affect the work of a class.
- 8.9 A teacher cannot remain impartial and is likely to favour certain students.
- 8.10 A student may consider himself victimised if he gets low marks consistently for what he considers is good work.
- 8.11 The re-orientation courses needed for teachers are expensive and time-consuming.
- 8.12 Lazy students copy assignments from their friends.
- 8.13 There is continuous work throughout the year even during the breaks and free time is limited.
- 8.14 With no external assessment too much burden is placed on teachers' opinions and they are subjected to pressure from the students.
- 8.15 Students are not encouraged to use their memories.
- 8.16 A teacher may pass too many students to improve his prestige and reputation.

- 8.17 Certain subjects like History and Language are not amenable to this approach.
- 8.18 Students from different institutions cannot be compared realistically.
- 8.19 There is no fixed standard for passing. This varies with the teacher and the institution.
- 8.20 Every teacher has to be scrupulously fair and honest.
- 8.21 A teacher feels that he ought to be paid more for running this system because of the extra work required.
- 8.22 A teacher needs a wide variety of new skills and the gaining of these puts too great a strain on the teacher.
- 8.23 Few places have libraries of sufficient extent to support the full implementation of the system.

*The tremendous amount of extra work required from both teacher and student is the limiting factor in the full implementation of the system. Only the full commitment of a teacher coupled with the goodwill of the students will get the system operational.*

## 9. HOW TO APPROACH A NEW COURSE

When faced with proposals for a new course the process is something like this:

- 9.1 What are the characteristics of the new course?
- 9.2 Are each of these characteristics essential - or just desirable?
- 9.3 *If the essential characteristics are impossible to foster through lack of funds, or public attitude, or whatever, the course has to be abandoned.*
- 9.4 If the essential characteristics are possible, which of the desirable characteristics should be included? Can enough of them be supported to make the course viable? *If YES, the course construction can go ahead. If NO, the course must be abandoned.*

## 10. HOW TO GET A NEW COURSE ACCEPTED

It is rarely easy to get a new course operational and supported by everyone. Three tactics will help.

- 10.1 INFORMATION on all aspects of the course must be freely available to all who will be in any way affected by its introduction.
- 10.2 Those who will have to operate the new course must be given appropriate EXPERIENCES from it in workshops and seminars whose methods closely resemble those of the new course.
- 10.3 Finally, the group who are to operate the new course must be the FINAL ARBITERS of its design. Unless the group norms of this set of people can be changed to acceptance of the system, it will not be implemented successfully.

It is the last step that is most frequently overlooked but change must come from within otherwise there is tremendous resistance and many problems. It is the job of the course designers to convince the users of the advantages of the new ideas. It is not good to rely on authority.

## 11. CONCLUDING REMARKS

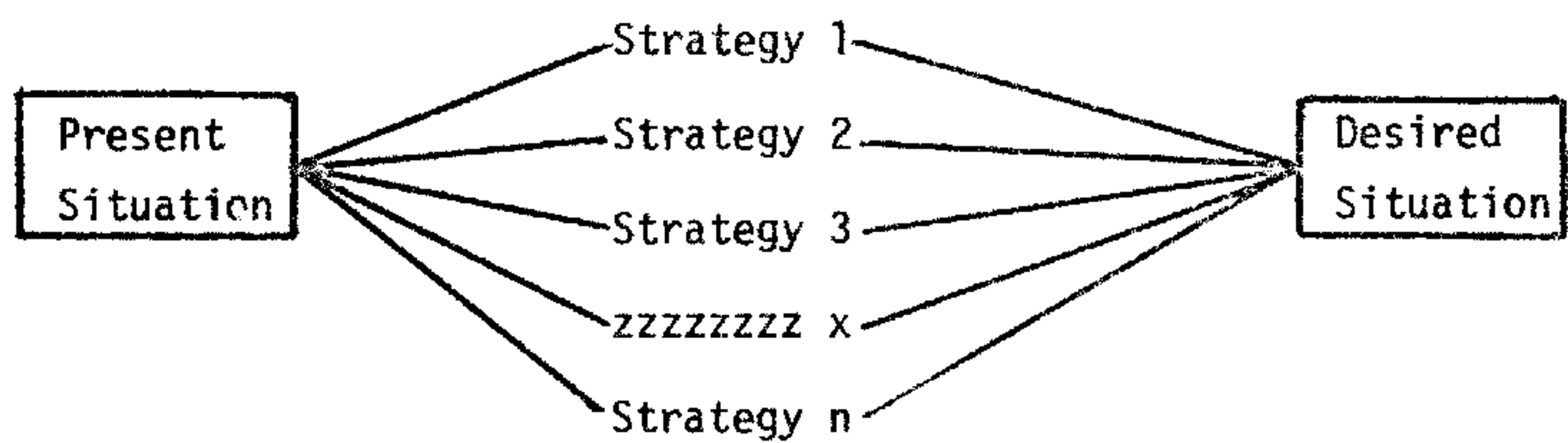
Every course has to be taken on its own merits and the nature of the course will determine the most suitable method of reaching the course objectives. In general, the more *academic* a course the more the need for lectures and external assessment with full-scale composite tests. The more *vocational* a course, the more the need for mastering the essential elements of the task.

In any case, each course should be custom-built for its task using the most up-to-date of suitable methodology without regard being paid for labels to attach to its structure.

[illegible]

### A Systems Approach

A systems approach to a problem can be represented simply as:



Both the present and the desired situation must be described in great detail.

The various strategies must be evaluated in terms of

- a. effectiveness
- b. efficiency

T.L.M.

26 Sep 81

# EVALUATION OF CURRICULUM CONTENT - OR A TEXTBOOK

DC/PAK/77/039/ED

Strengthening Selected Educational  
Institutions in N.W.F.P.

|                                                                  | POOR                     | FAIR                     | GOOD                     |
|------------------------------------------------------------------|--------------------------|--------------------------|--------------------------|
| 1. CONTENT                                                       |                          |                          |                          |
| 1.1 The items are those which fit the objectives of the course   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 1.2 The items are:                                               |                          |                          |                          |
| 1.2.1 interesting                                                | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 1.2.2 of correct difficulty                                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 1.2.3 useful                                                     | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 1.3 The items cover the official syllabus                        | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2. SUBJECT SOUNDNESS                                             |                          |                          |                          |
| 2.1 There are no subject errors                                  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2.2 The development is logically sound                           | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2.3 It is suitably precise                                       | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2.4 It is suitably rigorous                                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 2.5 All symbols and abbreviations are in the accepted convention | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3. LANGUAGE                                                      |                          |                          |                          |
| 3.1 The text is:                                                 |                          |                          |                          |
| 3.1.1 easily read (has simple constructions)                     | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.1.2 easily understood                                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.2 Complex situations are introduced in a simple way            | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.3 All symbols are carefully explained                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.4 The language creates interest                                | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.5 Questions are used to provoke deep thought                   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 3.6 The questions are at the level of the reader                 | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |



|                                                | POOR                     | FAIR                     | GOOD                     |
|------------------------------------------------|--------------------------|--------------------------|--------------------------|
| 4. EDUCATION                                   |                          |                          |                          |
| 4.1 The material promotes learning             | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.2 It is relevant to the learner's experience | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.3 Individual differences are allowed for     | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.4 The language fits that of the subject      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5 The language involves:                     |                          |                          |                          |
| 4.5.1 reflective thinking                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5.2 problem solving                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5.3 experimentation                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5.4 analysis                                 | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5.5 generalisation                           | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 4.5.6 self-evaluation                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5. MASTERY                                     |                          |                          |                          |
| 5.1 The exercises involve:                     |                          |                          |                          |
| 5.1.1 manipulative skills                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.1.2 reflective thinking                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.1.3 problem solving                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.2 Sufficient exercises are given             | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.3 Exercises are well graded                  | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.4 Review material is plentiful               | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.5 Remedial material is provided              | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.6 The exercises involve:                     |                          |                          |                          |
| 5.6.1 generalisations                          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.6.2 consolidating concepts                   | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.6.3 drill                                    | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 5.6.4 application                              | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6. ENRICHMENT                                  |                          |                          |                          |
| 6.1 Enrichment material is provided            | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6.2 References are given for                   |                          |                          |                          |
| 6.2.1 independent study                        | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6.2.2 research                                 | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6.2.3 projects                                 | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 6.2.4 extra reading                            | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 7. LEARNING AIDS                               |                          |                          |                          |
| 7.1 Worked solutions are provided              | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 7.2 Audio-visual aids are provided             | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 7.3 Teachers guides are provided               | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

|                                          | POOR                     | FAIR                     | GOOD                     |
|------------------------------------------|--------------------------|--------------------------|--------------------------|
| 8. PHYSICAL CHARACTERISTICS              |                          |                          |                          |
| 8.1 The layout is attractive             | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.2 The material is made readable by:    |                          |                          |                          |
| 8.2.1 layout                             | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.2.2 headings                           | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.2.3 second colour                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.2.4 style of type                      | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.2.5 size of type                       | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.3 Colour is used functionally          | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.4 Diagrams show concepts clearly       | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.5 It is a standard convenient size     | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |
| 8.6 It is well stitched and hard wearing | <input type="checkbox"/> | <input type="checkbox"/> | <input type="checkbox"/> |

## How to Use the Check List

1. The check list can be used either with a piece of content or with an actual textbook.
2. Each criterion can be marked as Poor, Fair, Good or Not Applicable.
3. If a score is needed for some comparison purpose, you might use poor - 1, fair - 2 and good - 3. In this case Not Applicable should be marked 2.

This would give a maximum of 168 and a minimum of 56 with an average of 112. Acceptable scores could be set for each section and for the whole piece of work; for example,

|       |       |       |         |       |
|-------|-------|-------|---------|-------|
| 1. 10 | 2. 10 | 3. 14 | 4. 20   | 5. 22 |
| 6. 10 | 7. 6  | 8. 20 | Overall | 112   |

Depending on the purpose of the piece of work some sections could be treated more leniently and others more rigorously.

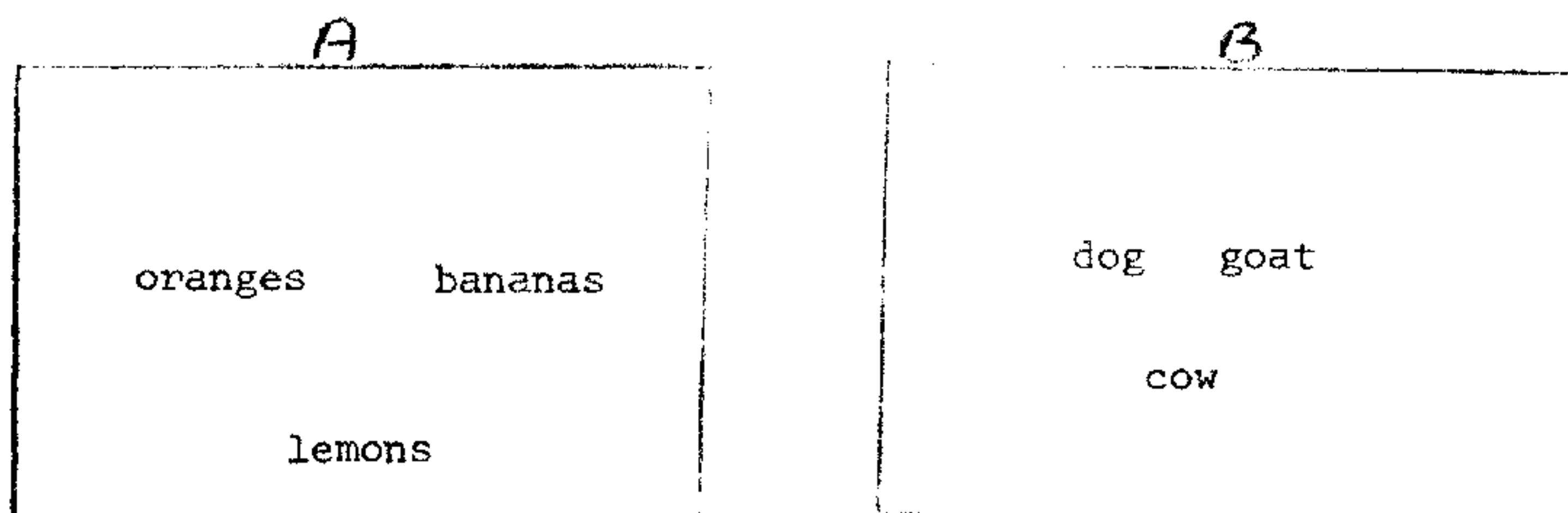
4. Some sections could also carry a power of veto. For example, if the items do not cover the official syllabus it may be completely useless.

# SETS FOR PRIMARY TEACHERS

DC/PAK/77/039/ED  
Strengthening Selected  
Educational Institutions  
in N.W.F.P.

## 1. ATTRIBUTES OF SETS

Look at these two pictures.



Into which picture would you put:

plum, horse, apple, mango, sheep, a pot, a cat, a chair?

We can call the things in Picture A a SET OF FRUITS.

What would you call the things in Picture B?

Plum, apple and mango were put into the first set because they were fruits. Horse, sheep and a cat were put into the second set because they were animals.

A pot and a chair could not be put into either of the two sets.

In Mathematics the things that make you decide to put objects into a set or not are the ATTRIBUTES of the object. A pot has neither the attributes of a fruit or an animal.

### Exercise:

1.1 Put the following objects into sets with the same attributes:

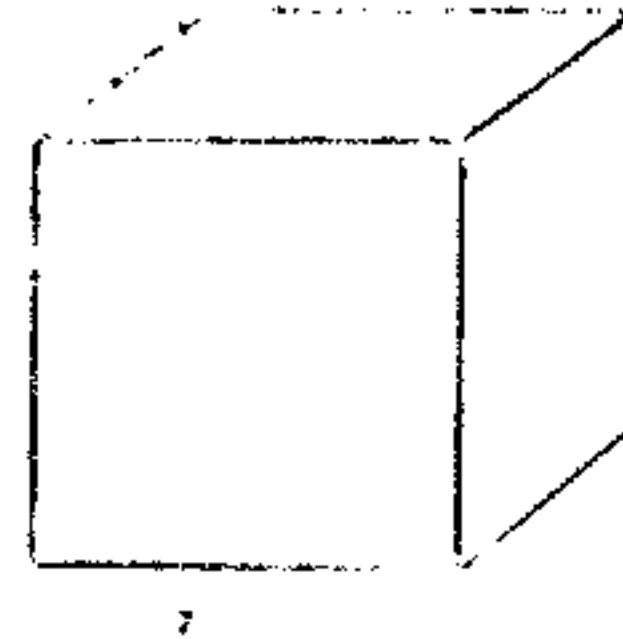
chair, car, table, bicycle, sitar, aeroplane, piano, ship, drum, guitar.

1.2 Put the following numbers into two sets so that each set has an obvious attribute. 1, 2, 3, 4, 5, 6, 7,

8, 9, 10. Name the attribute of each set.

## 2. MATHEMATICAL ATTRIBUTES

Complete this list of attributes for the object whose drawing is here.



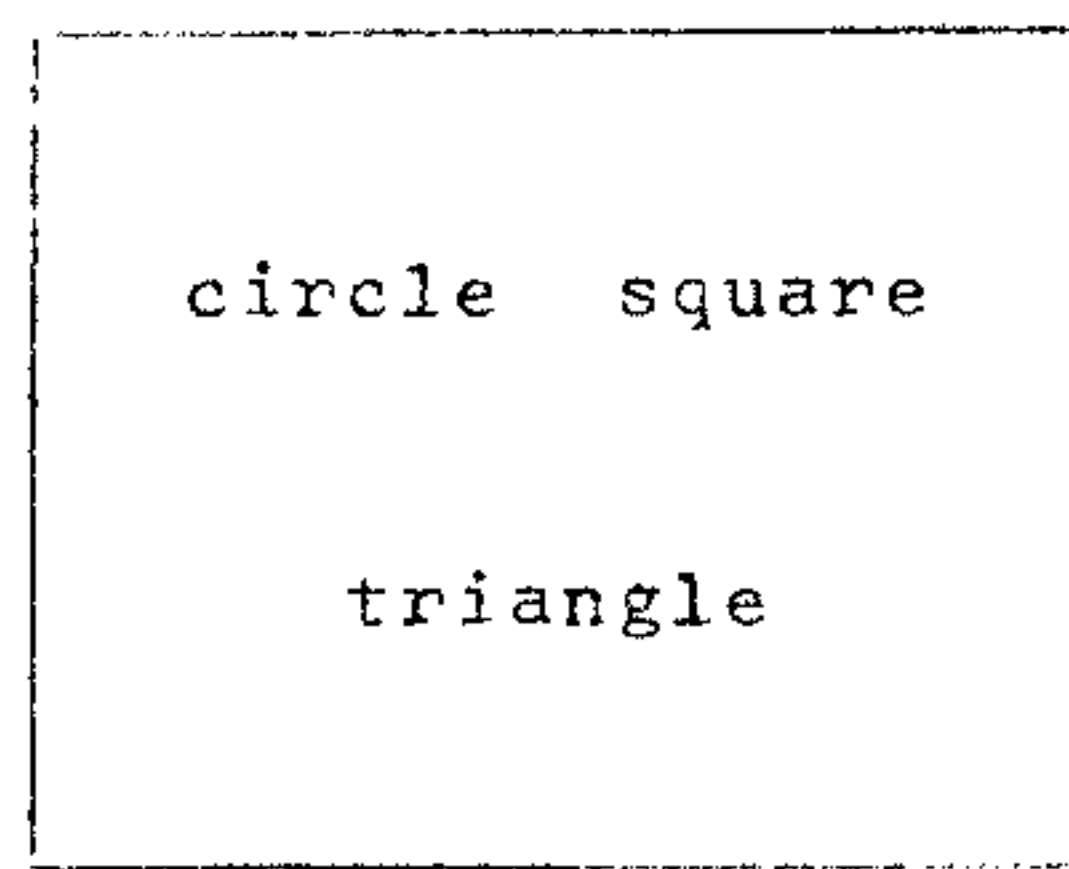
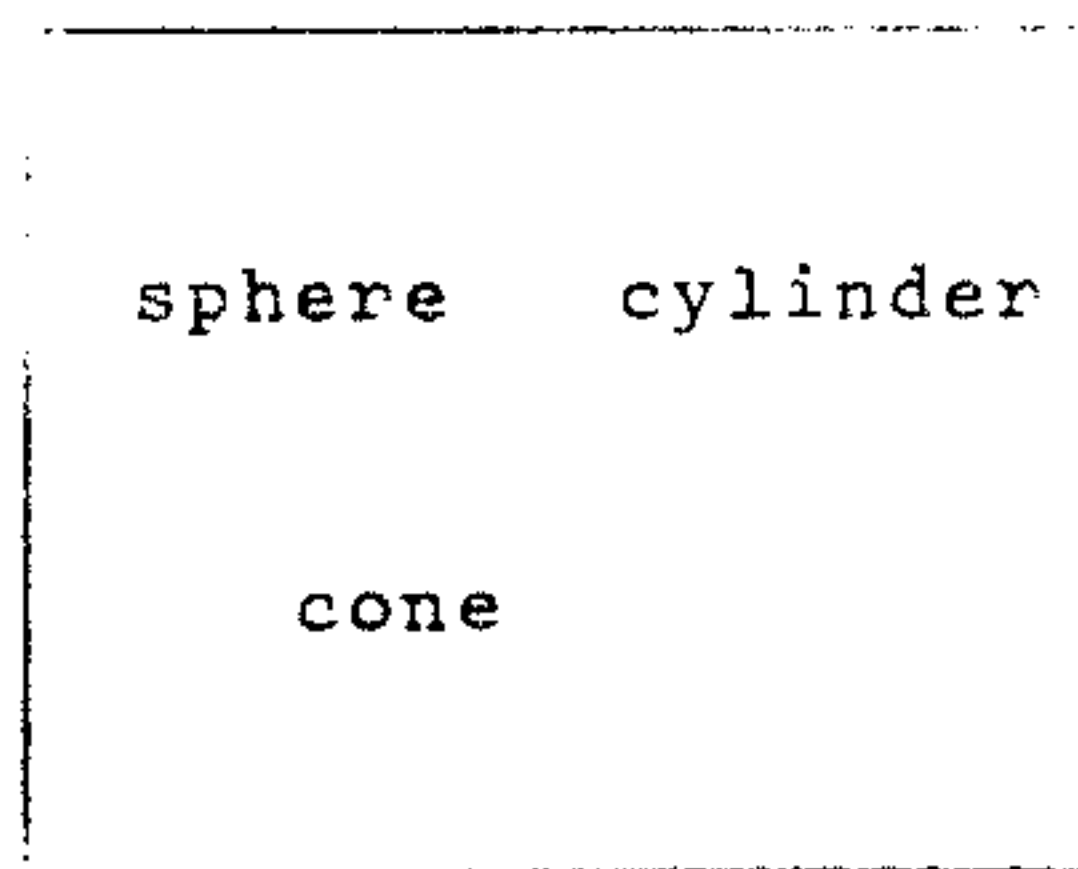
It is a cube.

Its colour is \_\_\_\_\_

It has \_\_ faces, \_\_ edges, and \_\_ corners.

Each face is the shape of \_\_\_\_\_

All these attributes are mathematical except one. The odd one out is colour because colour is not a mathematical attribute.



In which of these sets would you place the cube?

It goes into the first set because one of the properties of a cube is that it is a solid.

## 3. MEANING OF SET.

A set is undefined. The best we can say is that it is a collection of objects with some attributes in common. We try to stick to mathematical attributes but in Classes I to V sets are often made up of familiar objects which are grouped in non-mathematical ways.

### Exercise

Here are some sets. Name them and give the attribute you use.

- 3.1 {Saturday, Sunday, ... Friday}
- 3.2 {2, 4, 6, ...}
- 3.3 {1, 3, 5, ...}
- 3.4 {3, 6, 9, ...}
- 3.5 {2, 3, 5, 7, ...}

## 4. ELEMENTS OF A SET

$A = \{\text{Saturday, Sunday, ... Friday}\}$   
 $B = \{\text{January, February, ... December}\}$   
 $C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

### Exercise

- 4.1 What are the elements of set A?
- 4.2 What are the elements of set B?
- 4.3 What are the elements of set C?

Elements of a set are also called members.

## 5. MEMBERSHIP SYMBOL

We have already used the symbol for a set. For example,  
 $A = \{\text{Saturday, Sunday, ... Friday}\}.$

A set is usually labelled with a capital letter like A.  
The elements are enclosed in braces,  $\{ \}$ .

The other grouping symbols which are not normally used  
for sets are brackets  $[ ]$ , and parenthesis  $( )$ .

The symbol used for "is an element of" is  $\in$ .  
For example, Tuesday  $\in A$ .

The symbol used for "is not an element of" is  $\notin$ .  
For example, October  $\notin A$ .



## Exercise

Complete these by writing  $\in$  or  $\notin$ .

- 5.1 Friday {days of the week}
- 5.2 January {days of the week}
- 5.3 4 {even numbers}
- 5.4 3 {even numbers}
- 5.5 11 {prime numbers}
- 5.6 litre {units of volume}
- 5.7 centimetre {units of area}
- 5.8 kilogram {units of distance}
- 5.9 1984 {leap year}

## 6. NUMBER OF ELEMENTS IN A SET

The number of elements in set A is written  $n(A)$ .

For example,  $B = \{a, e, i, o, u\}$   $n(B) = 5$ .

The number of elements in a set is its CARDINALITY.

It is important not to confuse the set with its cardinality. In the primary work we often see things like:

$$\{ @, \#, \$, \& \}$$

4

It should be written clearly,  $n(\{ @, \#, \$, \& \}) = 4$

## 7. SOME TYPES OF SETS

### 7.1 FINITE SETS

Sets in which the number of elements are known or could be counted are called finite sets. The number of elements can vary from very few to very many.

{The pupils in a class} is a finite set with a relatively small membership.

{The people of N.W.F.P.} is a large finite set.

## 7.2 VERY LARGE SETS

Some sets are so large that it is hard to say how many elements they contain. Examples are {the grains of sand in N.W.F.P.} and {counting numbers}.

## 7.3 INFINITE SETS

Although {the grains of sand in N.W.F.P.} and {counting numbers} are both very large sets they are different. There is a definite number of grains of sand and we can imagine easily taking some away or adding to them and so changing the size of the set. The set of grains is finite although it would be hard to count them.

With counting numbers we cannot add any more to the set. They are all there already. This type of set is infinite.

It is an unfortunate fact that most of the sets we deal with in Mathematics are infinite. Probably all that a primary child must really know is that with an infinite set you can never write the last one. No matter what anyone says you can always add at least one more.

## 7.4 The EMPTY SET

A set is empty if it has no elements in it. It is written {} or  $\emptyset$  (phi). The empty set is common but the problem is that there is only one empty set. You cannot have AN empty set. Starting with 3 pencils and going down 2 pencils, 1 pencil 0 pencils gets you to the same empty set as going down 3 cows, 2 cows, 1 cow, 0 cows.

Care is also needed with {0}. This has one element.

## 7.5 UNIVERSAL SETS

For any given situation, it is important to know the basic set from which the elements that are to be used have to be taken. For example, if a problem concerns prime numbers less than 20 you only need to be concerned with counting numbers under 20. This would be the Universal set for the problem.

School Algebra has a Universal set for  $x$  of real numbers unless another restriction is applied.

For example, 'solve  $2x = 9$ , where  $x$  is a counting number.'

The 'solution' is  $x = 4\frac{1}{2}$  but this is not a counting number. The solution set is  $\emptyset$ .

The usual symbols for the Universal set are  $E$  or  $U$ , although  $E$  seems the most common in Pakistan.

In the example we had above:

$$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19\}$$

$$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$$

## 8. THE COMPLEMENT OF A SET

When a set is chosen from a Universal set what is left over is the complement of the chosen set. In the example of the last section, the complement of  $P$ , written  $P' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18\}$

For whole numbers, the complement of the set of odd numbers is the set of even numbers.

## Exercise

Classify each of these sets as finite, infinite or empty.

- 8.1. {days of the week}
- 8.2. {five-headed men}
- 8.3. {drops of water}
- 8.4. {houses in Peshawar}
- 8.5. {cars in N.W.F.P.}
- 8.6. {odd numbers in 2, 4, 6, 8}
- 8.7. {stars in the sky}
- 8.8. {odd numbers}
- 8.9. {points on a line segment}
- 8.10. {fractions between 0 and 1}

Take  $E = \{a, b, c, d, e, f, g, h, i, j\}$

$A = \{b, d, f, h, j\}$

$B = \{c, f, i\}$

$C = \{d, h\}$

$D = \{a, c, e, g, h, i\}$

- 8.11 List the members of  
 $A', D', B'$ .

- 8.12 Find  $n(E'), n(C'), n(E)$

- 8.13 Are these true or false?

$$n(\emptyset) = 0, A' = D, n(B) + n(C) = A, n(A) + n(D) = n(E)$$

- 8.14 If  $E = \{\text{integers from 1 to 10}\}$  and  
 $P = \{\text{odd integers from 1 to 10}\}$   
find  $P'$  and  $n(P) + n(P')$ .

## 9. SUBSETS

$A = \{\text{Saturday, Sunday, ... Friday}\}$

$B = \{\text{Friday, Saturday}\}$

$C = \{\text{Monday, Tuesday, Wednesday}\}$

The sets B, C, and D are made up of elements from A so they are SUBSETS of A.

If  $p = \{1, 2, 3, 4\}$  the following subsets can be made:

$\{1, 2, 3\}$   $\{1, 2, 4\}$   $\{1, 3, 4\}$   $\{2, 3, 4\}$

$\{1, 2\}$   $\{1, 3\}$   $\{1, 4\}$   $\{2, 3\}$   $\{2, 4\}$   $\{3, 4\}$

$\{1\}$   $\{2\}$   $\{3\}$   $\{4\}$

$\emptyset$  (None of the elements are chosen.  $\emptyset$  is a subset of all sets)

$\{1, 2, 3, 4\}$  (All the elements have been chosen. The set itself is always a subset)

### Exercise

9.1 Write out all the possible subsets of  $\{\text{days of the week}\}$ .

9.2  $K = \{a, b, c\}$ . Write all the subsets of K.

9.3 Complete this table:

| SET              | NUMBER OF<br>ELEMENTS | NUMBER OF<br>SUBSETS |
|------------------|-----------------------|----------------------|
| $\emptyset$      |                       |                      |
| $\{1\}$          |                       |                      |
| $\{1, 2\}$       |                       |                      |
| $\{1, 2, 3\}$    |                       |                      |
| $\{1, 2, 3, 4\}$ |                       |                      |

Express the number of subsets as powers of 2.

How many subsets has a set of

a. 5 elements      b. 8 elements      c. n elements?

9.4 The symbol for subset is  $\subset$ . Classify as true or false:

- $\{\text{cats}\} \subset \{\text{animals}\}$
- $\{\text{Pakistanis}\} \supset \{\text{Asians}\}$
- $\{\text{vowels}\} \subset \{\text{letters of the alphabet}\}$
- $\{1, 2, 3\} \supset \{1, 2, 3, 4, 6\}$

## 10. SETS AND SUBSETS

When a set is taken from a Universal set it is called a set. When it is taken from another set it is called a subset.

In the last section we took  $A = \{\text{days of the week}\}$  as a set and made subsets from it.

If we had started by taking the Universal set  $E = \{\text{days of the week}\}$ ,  $\{\text{Friday, Saturday}\}$  and  $\{\text{Monday, Tuesday, Wednesday}\}$  would have been referred to as sets, not subsets.

## 11. DISJOINT SETS

The sets  $A = \{a, b, c, d\}$  and  $B = \{x, y, z\}$  do not have any elements in common and so they are called disjoint sets.

If  $C = \{a, c, e\}$  then  $A$  and  $C$  are not disjoint.

In Classes I to V, where we often refer to concrete sets, we need to take some care. We take a set of pencils and put it together with another set of pencils. We treat the sets as if they were disjoint with no elements in common but they both have pencils.

This difficulty can only be got over if we think of each pencil as unique. Each one has to be different; pencil 1, pencil 2 and so on.

Each of these two sets contains a pencil.



However, we treat the sets as disjoint UNLESS it is the SAME pencil in each set



## 12. THE UNION OF SETS

Given  $A = \{\text{Sunday, Monday}\}$  and

$B = \{\text{Tuesday, Wednesday, Thursday}\}$

then the UNION of sets A and B is the set

$C = \{\text{Sunday, Monday, Tuesday, Wednesday, Thursday}\}$

The symbol for union is  $\cup$ , so  $A \cup B = C$ .

If  $D = \{\text{Wednesday, Thursday, Friday}\}$

$B \cup D = \{\text{Tuesday, Wednesday, Thursday, Friday}\}$

Common elements are not repeated.

This is the reason why, in the primary school, the union of sets is restricted to disjoint sets. Only in that way can union and addition be linked.

$n(B) = 3$      $n(D) = 3$     but  $n(B \cup D) = 4$  not 6

### Exercise

12.1     $A = \{1, 2, 3\}$      $B = \{2, 3, 4\}$      $C = \{4, 5\}$

Find a.  $A \cup B$

b.  $A \cup C$

c.  $B \cup C$

12.2     $E = \{1, 2, 3, 4, 5, 6\}$

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5\}$

Find a.  $A \cup B$

b.  $A \cup B'$

c.  $A' \cup B$

d.  $A \cup E$

e.  $B \cup A$

f.  $A' \cup B'$

g.  $B' \cup E$

### 13. THE INTERSECTION OF SETS

Given  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5\}$  then the INTERSECTION of A and B is the set  $C = \{3, 4\}$

The symbol for intersection is  $\cap$ , so  $A \cap B = C$

$P = \{a, b, c\}$  and  $Q = \{d, e, f\}$   $P \cap Q = \emptyset$

The intersection of disjoint sets is the empty set.

#### Exercise

Let  $E = \{\text{integers from 1 to 16}\}$

$A = \{\text{even integers}\}$        $B = \{\text{multiples of 4}\}$

$C = \{\text{perfect squares}\}$        $D = \{\text{multiples of 6}\}$

Find:

13.1  $A \cap B$

13.6  $B \cap A$

13.2  $C \cap A$

13.7  $E \cap C$

13.3  $A' \cap C$

13.8  $A \cap A'$

13.4  $B' \cap D'$

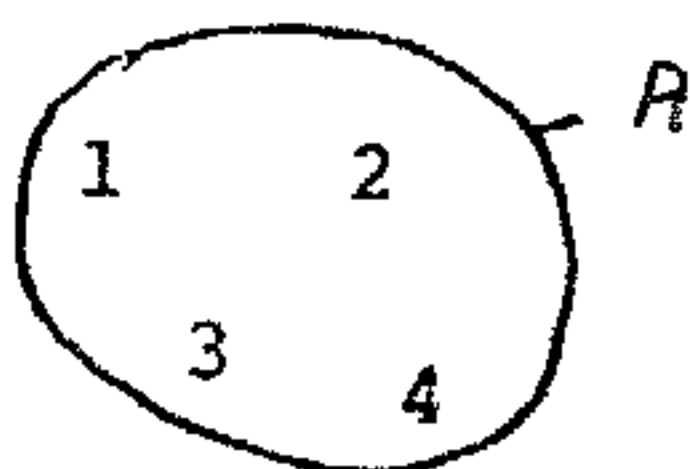
13.9  $A \cap (B \cap C)$

13.5  $C \cap D$

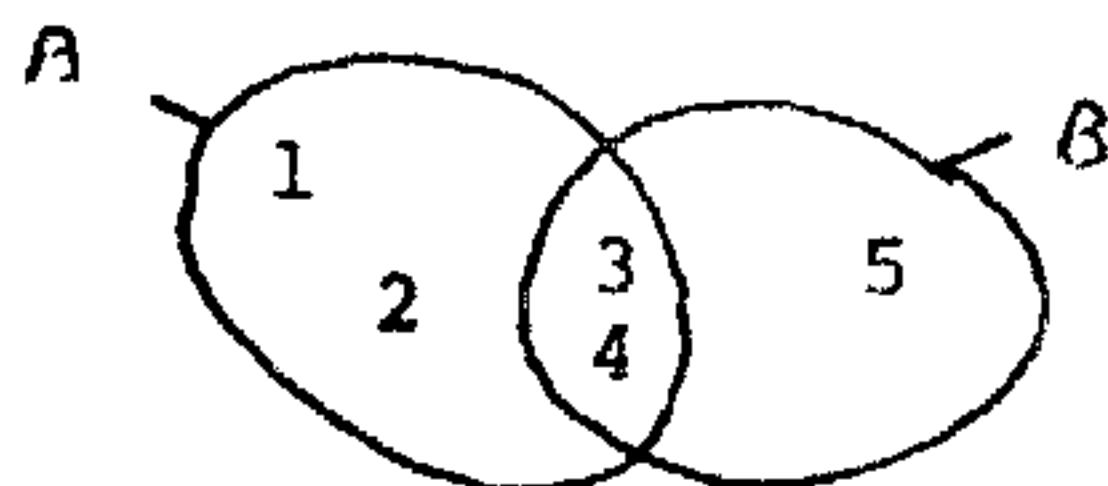
13.10  $(A \cap B) \cap C$

### 14. VENN DIAGRAMS

A set  $A = \{1, 2, 3, 4\}$  is often drawn like this:

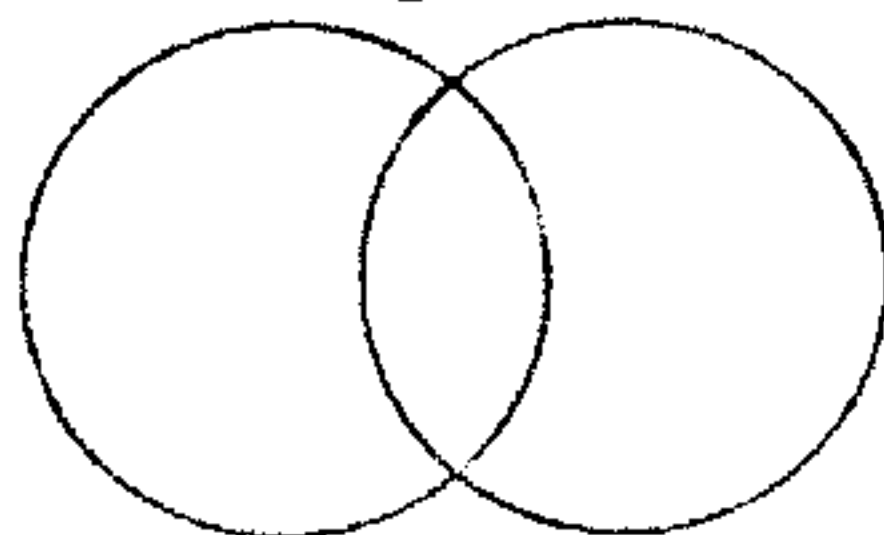


Another set  $B = \{3, 4, 5\}$  can be added to the sketch

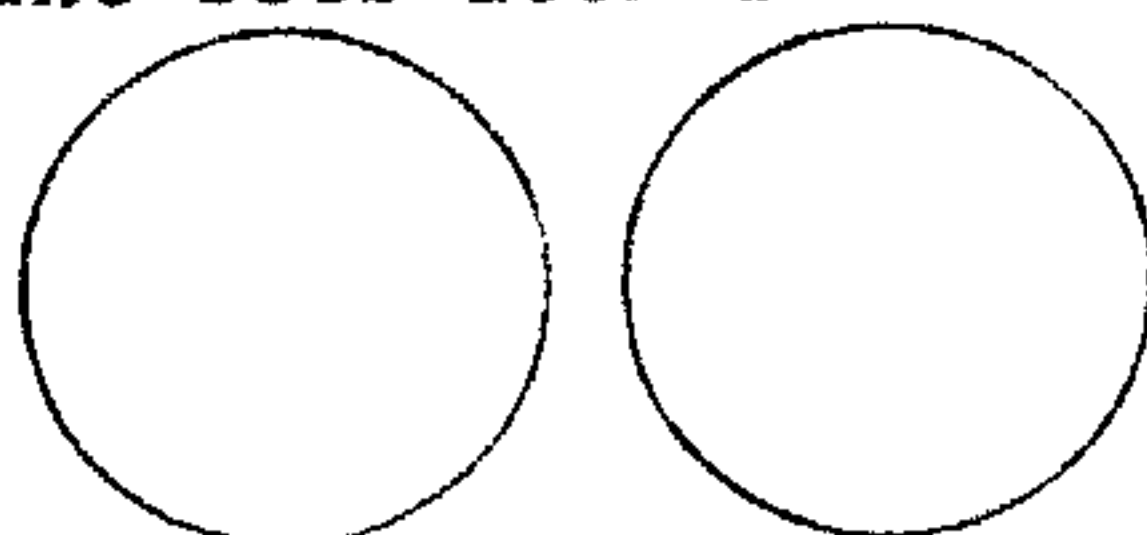


Such a diagram is a Venn diagram.

Two intersecting sets always look like this:



Two disjoint sets look like:



Exercise

14.1 Show on a Venn diagram

$$P = \{1, 2, 3, 4, 5, 6\} \text{ and } Q = \{2, 4, 6, 8, 10\}$$

14.2 Illustrate:

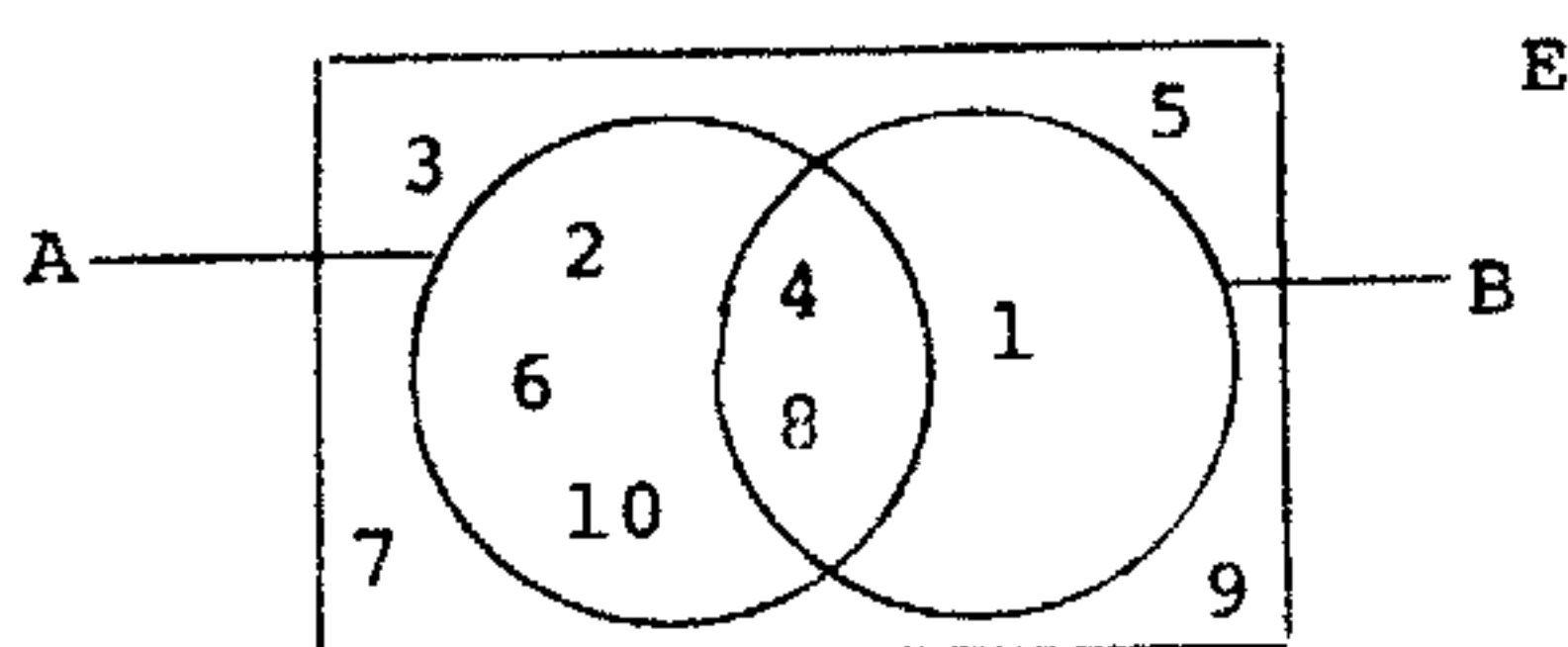
$$S = \{a, e, i, o, u\} \text{ and } T = \{b, d, c, d, f\}$$

## 15. VENN DIAGRAMS AND UNIVERSAL SETS

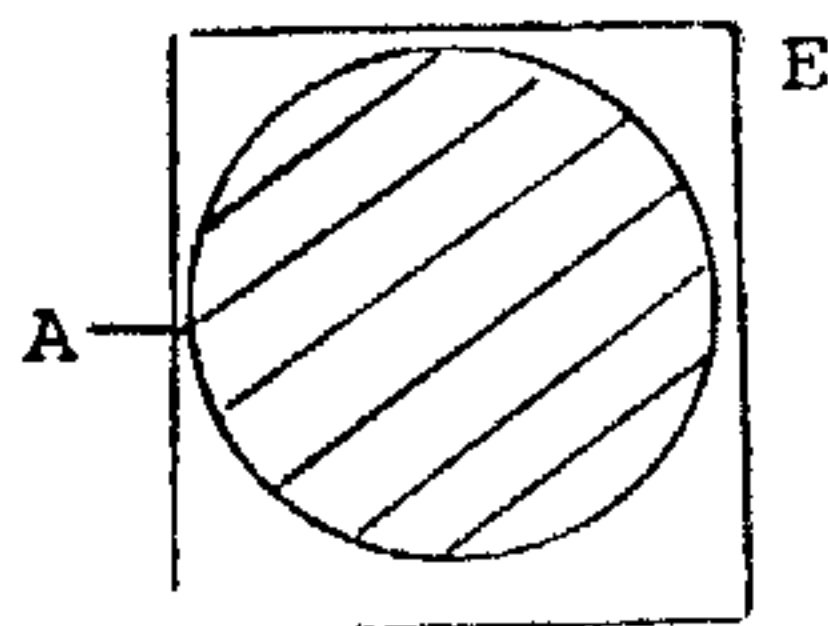
$$E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 4, 6, 8, 10\} \quad B = \{1, 4, 8\}$$

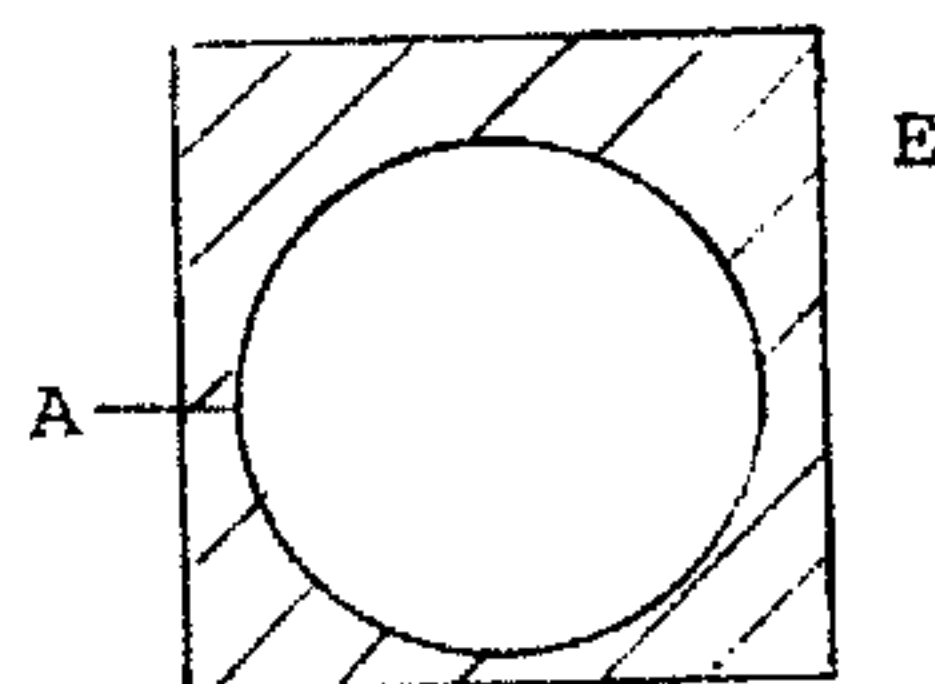
The Venn diagram for this is:



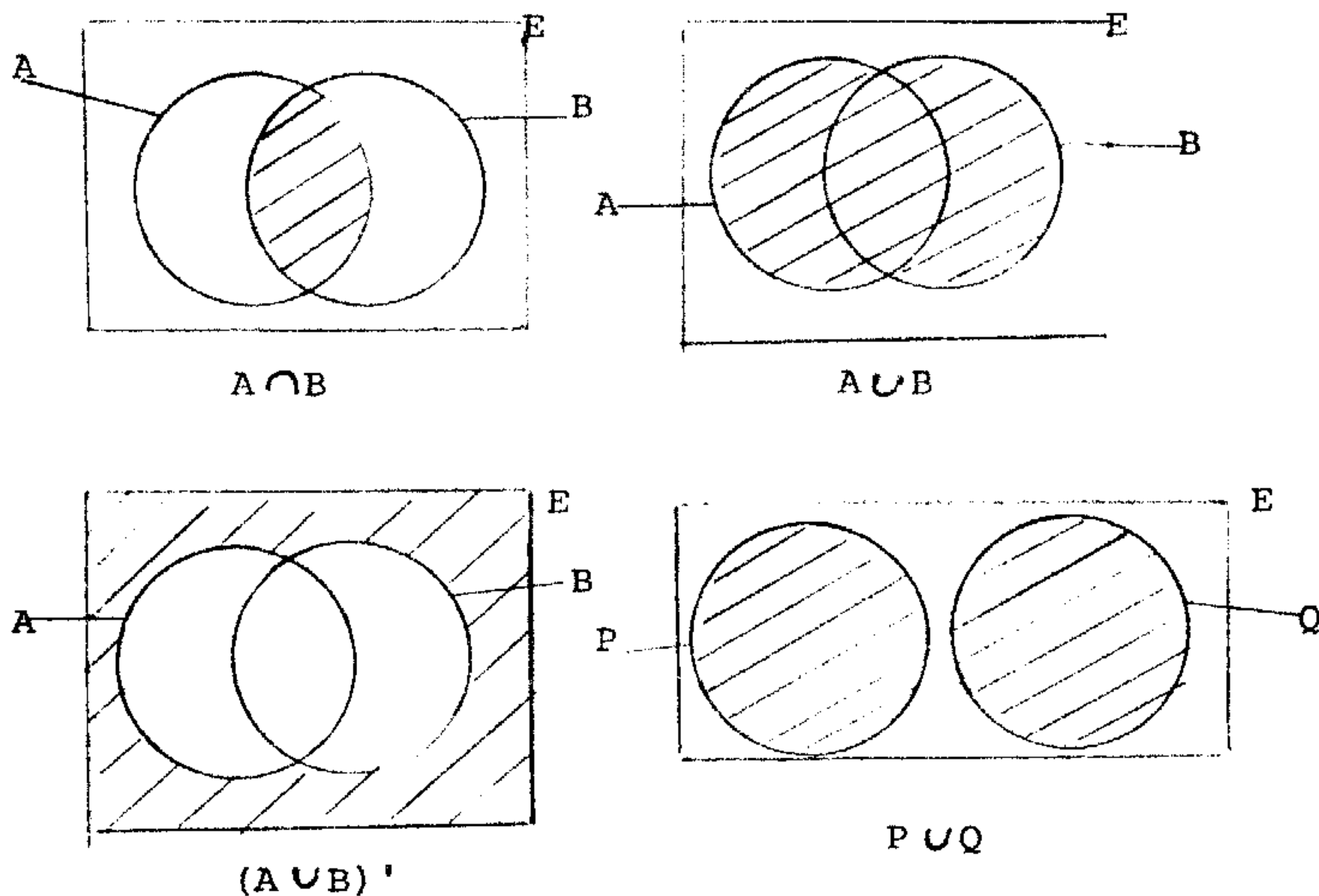
In general we have diagrams like these:



A

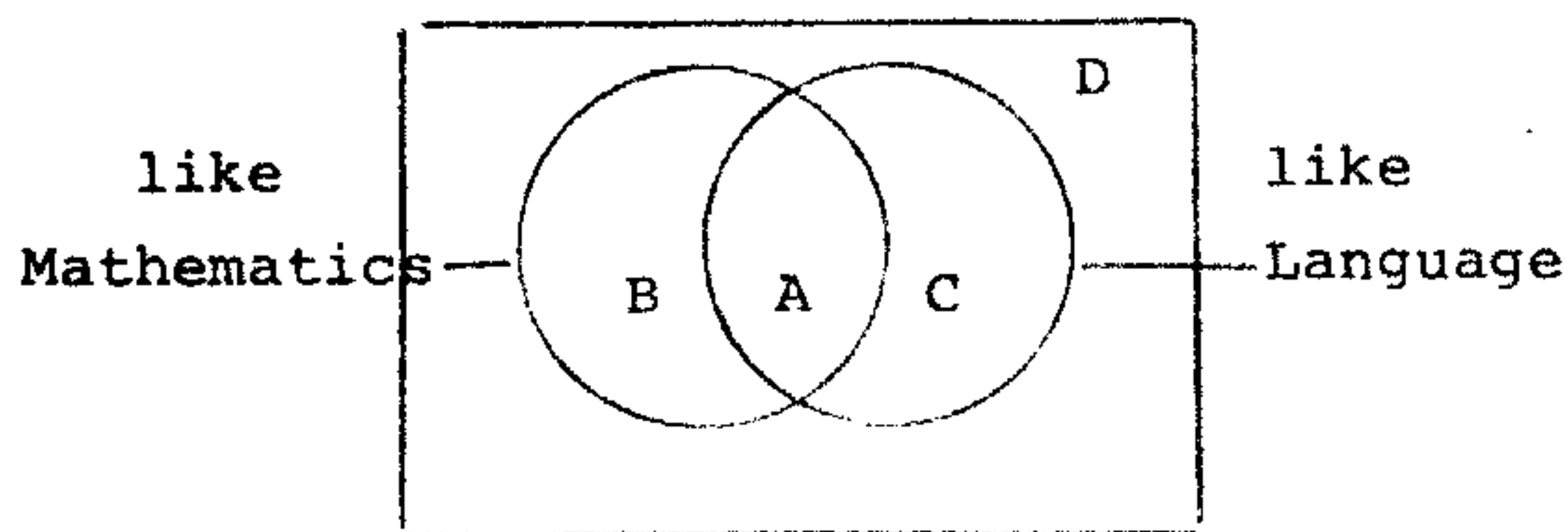


A'



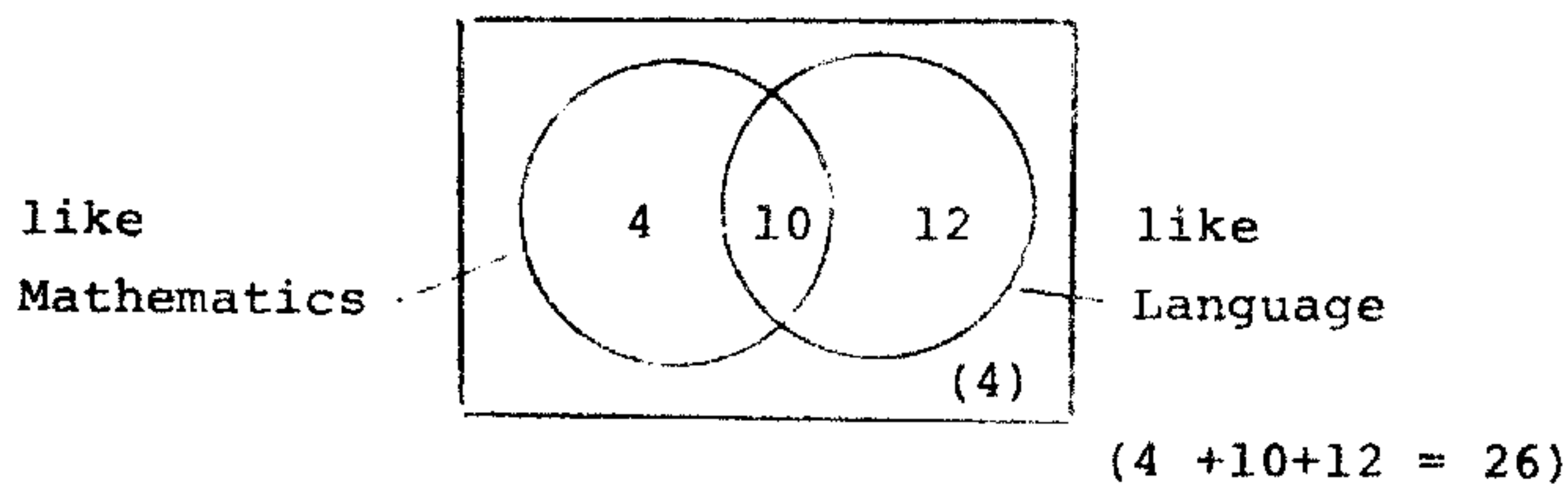
## 16. SOLVING PROBLEMS WITH VENN DIAGRAMS

In a class of 30 children, 10 like Mathematics and Language, 4 like Mathematics only and 12 like Language only. How many do not like Mathematics or Language? This is easily done with a Venn diagram.



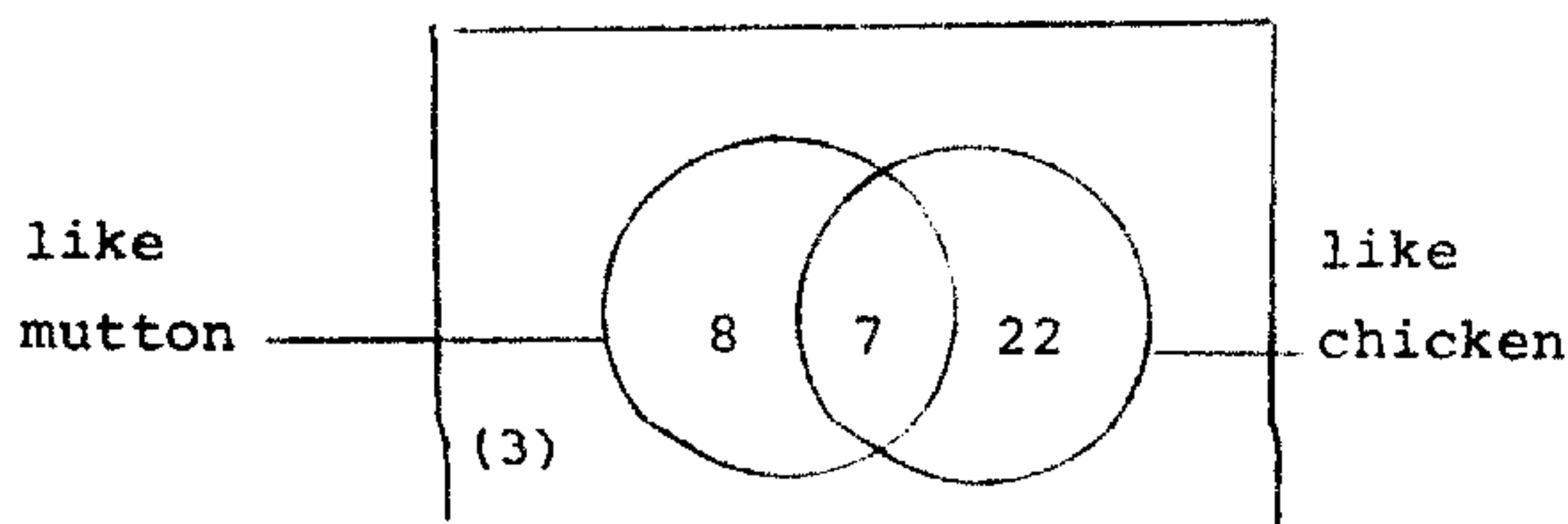
- Region A - like Mathematics and Language
- Region B - like Mathematics but not Language
- Region C - like Language but not Mathematics
- Region D - like neither Mathematics or Language

Putting figures in the appropriate regions gives:



4 pupils do not like Mathematics or Language.

In a group of 40 people, 15 like mutton, 29 like chicken and 7 like both. How many like neither?  
Here is the solution:



The first figure to be entered is the 7 in the intersection.

Since 15 like mutton, that leaves 8 to go in the mutton only region.  $(15 - 7 = 8)$

Since 29 like chicken, 22 like chicken only.  $(29 - 7 = 22)$

$$8 + 7 + 22 = 37 \quad 40 - 37 = 3$$

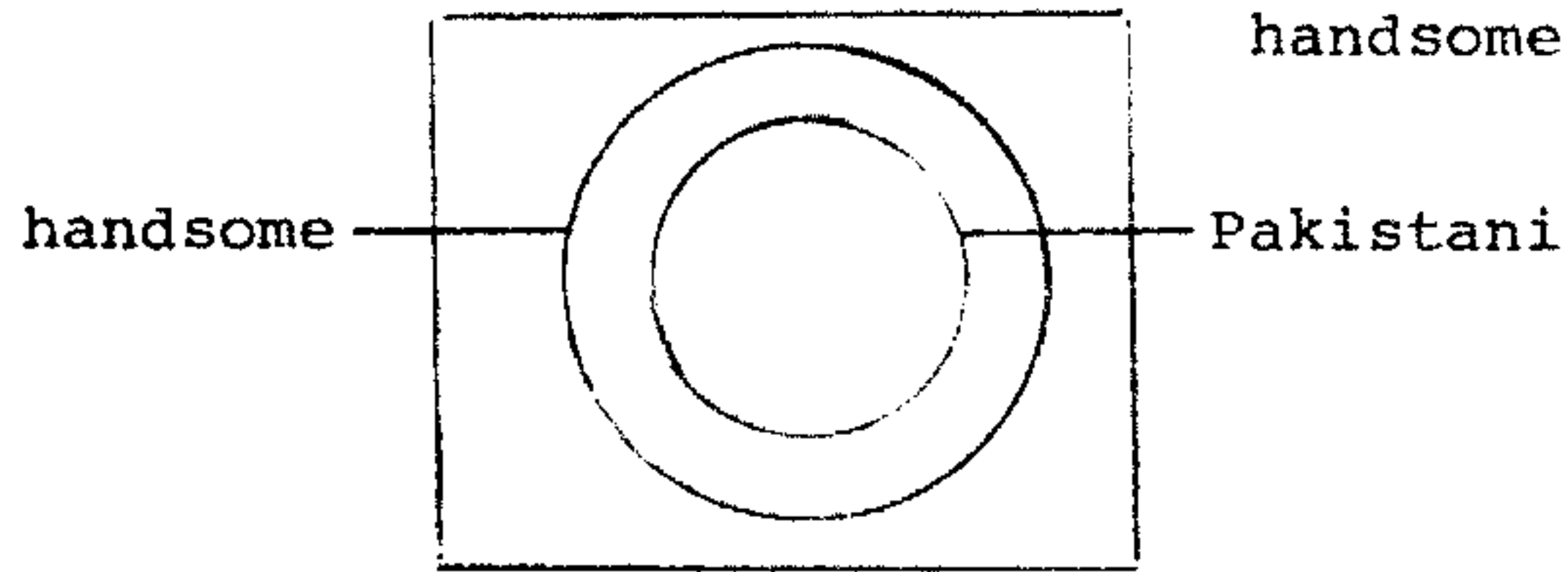
Three people like neither mutton or chicken.

Venn diagrams can also be used to solve logical problems.

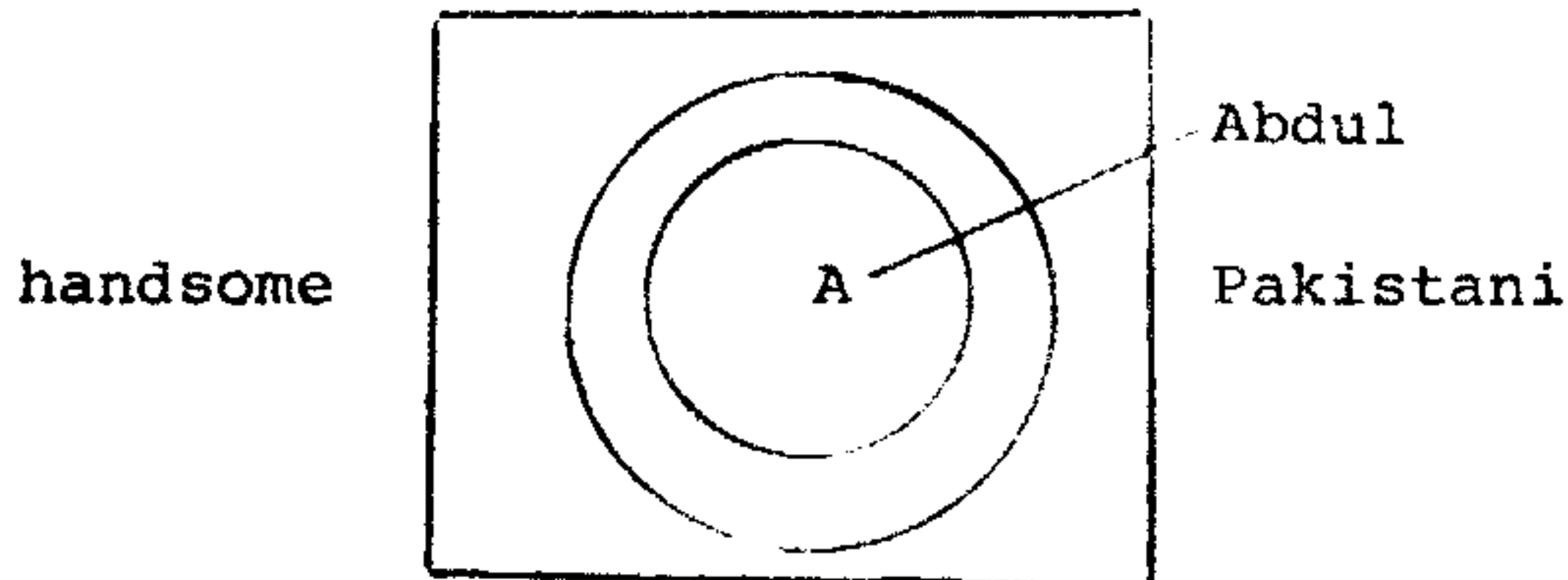
Here is a syllogism: All Pakistanis are handsome  
Abdul is a Pakistani  
Thus Abdul is handsome.

Is this valid or invalid?

Here is the Venn diagram for all Pakistanis are handsome



Where does Abdul appear in the diagram?



The conclusion is valid.

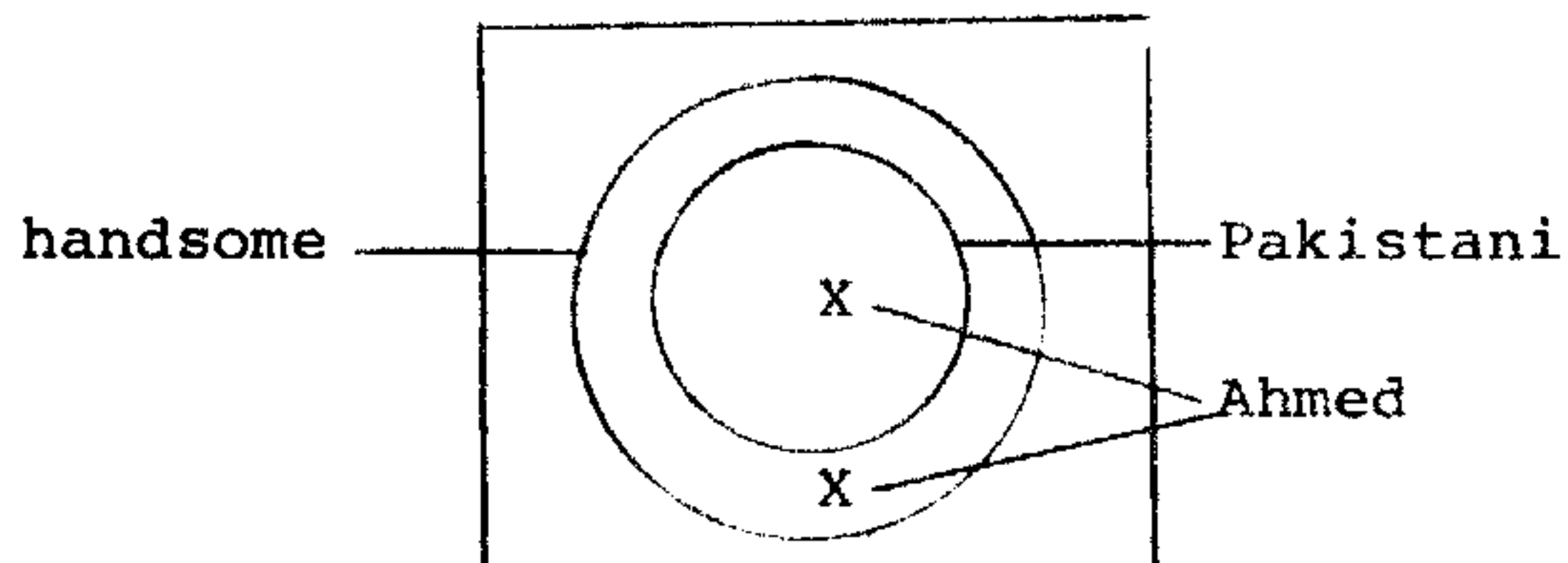
Compare this to:

All Pakistanis are handsome

Ahmed is handsome

Thus Ahmed is a Pakistani.

Is this valid? Here is the diagram.



The conclusion is invalid. He may or may not be Pakistani. There are two regions where he may be.



### Exercise

1. A group of 300 children were asked if they liked coca-cola or 7-up. 120 liked coca-cola only and 70 liked 7-Up only. 25 liked neither.
  - a. How many liked coca-cola?
  - b. How many liked 7-Up?
2. All dogs bark. Rover barks. Thus Rover is a dog. Is this valid?
3. Some teachers have beards. All men with beards are intelligent. Iqbal is an intelligent teacher so he must have a beard. Is this valid?
4. 50 people were asked if they like rice and potatoes. 26 said they liked both. 15 liked rice but not potatoes. 1 person liked potatoes but not rice.
  - a. How many liked neither?
  - b. How many liked rice?
  - c. How many liked potatoes?
5. 100 people were asked if they liked cricket or hockey. 72 liked both and 6 liked neither. If 82 people liked cricket, how many liked hockey?

#####

DC/PAK/77/039/ED  
Strengthening Selected Educational  
Institutions in N.W.F.P.

NUMBER SETS AND NUMBER LAWS IN THE FIRST FIVE  
CLASSES OF THE PRIMARY SCHOOLS IN N.W.F.P.

BUREAU OF CURRICULUM DEVELOPMENT. ABBOTTABAD, N.W.F.P.  
UNITED NATIONS EDUCATIONAL SCIENTIFIC AND CULTURAL ORGANISATION  
NOVEMBER 1981

## O B J E C T I V E S

THIS UNIT IS DESIGNED TO FORMALISE THE TEACHERS' KNOWLEDGE OF NUMBER THEORY AND LAWS FOR THE EFFECTIVE TEACHING OF NUMERACY.

AFTER COMPLETING THE UNIT, THE TRAINEE WILL BE ABLE TO:

1. LIST THE SET OF NUMBERS
  - 1.1 NATURAL NUMBERS
  - 1.2 WHOLE NUMBERS
  - 1.3 INTEGERS
  - 1.4 POSITIVE INTEGERS
  - 1.5 NEGATIVE INTEGERS
  - 1.6 RATIONAL NUMBERS
2. EXPLAIN
  - 2.1 FRACTIONS
  - 2.2 DECIMAL FRACTIONS
  - 2.3 RATIONAL NUMBERS
3. IDENTIFY THE SYMBOLS FOR NUMBER SETS
  - 3.1  $\mathbb{N}$
  - 3.2  $\mathbb{W}$
  - 3.3  $\mathbb{Z}$
  - 3.4  $\mathbb{Z}^+$
  - 3.5  $\mathbb{Z}^-$
  - 3.6  $\mathbb{Q}$
4. DISTINGUISH
  - 4.1 A NUMBER FROM A NUMERAL
  - 4.2 A STATE FROM AN OPERATION
  - 4.3 PLUS FROM POSITIVE
  - 4.4 MINUS FROM NEGATIVE
5. LIST THE FOUR OPERATIONS OF ARITHMETIC
6. EXPLAIN
  - 6.1 ADDITION
  - 6.2 SUBTRACTION
  - 6.3 MULTIPLICATION
  - 6.4 DIVISION

7. SHOW DIAGRAMS TO ILLUSTRATE
  - 7.1 ADDITION
  - 7.2 SUBTRACTION
  - 7.3 MULTIPLICATION
  - 7.4 DIVISION
8. NAME MATHEMATICAL SYSTEMS COMPRISING  
NUMBER SETS AND THE FOUR OPERATIONS
9. DEFINE
  - 9.1 CLOSURE
  - 9.2 COMMUTATIVE
  - 9.3 ASSOCAITIVE
10. GIVE POSITIVE AND NEGATIVE EXAMPLES OF  
SYSTEMS WHICH ARE
  - 10.1 CLOSED
  - 10.2 COMMUTATIVE
  - 10.3 ASSOCIATIVE
11. TEST ANY GIVEN SYSTEM FOR
  - 11.1 CLOSURE
  - 11.2 COMMUTATIVITY
  - 11.3 ASSOCIATIVITY
12. DEFINE AN IDENTITY ELEMENT
13. FIND THE IDENTITY ELEMENT OF ANY GIVEN  
SYSTEM
14. DEFINE
  - 14.1 AN INVERSE OPERATION
  - 14.2 AN INVERSE ELEMENT
15. FIND THE INVERSE OPERATION OF A GIVEN  
OPERATION
16. FIND INVERSE ELEMENTS IN A GIVEN SYSTEM
17. DEFINE THE DISTRIBUTIVE LAW
18. GIVE POSITIVE AND NEGATIVE EXAMPLES OF  
THE DISTRIBUTIVE LAW
19. TEST A GIVEN SYSTEM OF A SET OF NUMBERS  
AND TWO OPERATIONS FOR DISTRIBUTIVITY
20. USE THE KNOWLEDGE GAINED TO TEACH NUMERACY  
EFFECTIVELY IN CLASSES I TO V.

# 1. NUMBER SETS

The first set of numbers that is usually met is the counting numbers. These are 1, 2, 3, 4, 5, and so on.

However, when we count the elements in a set we include the empty set so there is some confusion whether 0 is a counting number or not. This is avoided if we talk about NATURAL NUMBERS and WHOLE NUMBERS.

The set of Natural Numbers is:  $N = \{1, 2, 3, 4, \dots\}$

The set of Whole Numbers is:  $W = \{0, 1, 2, 3, \dots\}$

Note that the three dots is a conventional symbol for an INFINITE SET which continues in the same pattern.

The set of natural numbers can also be called the set of POSITIVE INTEGERS, although in this case the symbol is  $Z^+$ .

The complete set of INTEGERS is made up of:

the positive integers,  $Z^+$

the negative integers,  $Z^-$  and

zero, 0.

The set of Integers is:  $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

ODD NUMBERS, EVEN NUMBERS, PRIME NUMBERS COMPOSITE NUMBERS, SQUARE NUMBERS and CUBIC NUMBERS are all SUBSETS of the set of natural numbers.

The only other sets of numbers that cause bother in the primary school are FRACTIONS and DECIMAL FRACTIONS.

It is not very wise to talk of THE SET OF FRACTIONS. The integers may or may not be written in fractional form.

2 is not a fraction.  $\frac{2}{1}$  is a fraction.

Fractions are not so much a type of number as a way of writing numbers.

What is needed is a set of numbers that includes integers, fractions and decimal fractions as elements.

2 can be written as  $\frac{2}{1}$ . 0.6 can be written as  $\frac{6}{10}$ .

This gives us the idea that we can have a set of numbers of the type  $\frac{a}{b}$ .

Where  $b = 1$  we get an integer.

Where  $b$  is a power of 10 we get a decimal fraction.

Other values of  $a$  and  $b$  give us fractions but what sets should  $a$  and  $b$  come from?

We take  $a$  from the set of integers and  $b$  from the set of natural numbers and call the set, the set of RATIONAL NUMBERS.

The set of Rational Numbers  $Q = \left\{ \frac{a}{b} \right\}$  where  $a \in \mathbb{Z}$  and  $b \in \mathbb{N}$

Examples of rational numbers are 5,  $\frac{3}{8}$ ,  $-2$ , 0, 0.34,  $-1\frac{1}{2}$ .

It is worth noting that this definition puts the sign with the numerator of a fraction.  $-\frac{4}{5}$  is  $-4$  over  $+5$ . This helps to avoid the error where a person thinks the negative sign is attached to both the numerator and denominator.

## 2. NUMBERS AND NUMERALS

A number is an abstract concept. A numeral is a way of writing that abstraction down.

This is the numeral 2. There is also a number 2 which is a property that pairs of objects have.

The numeral 2 can be written in lots of ways and can have lots of different names in different languages but the abstract number 2 stays the same.

It is common to stress this distinction with children but the important thing is that the teacher keeps the concepts in mind and says 'number' when he mean number and 'numeral' when he means numeral.



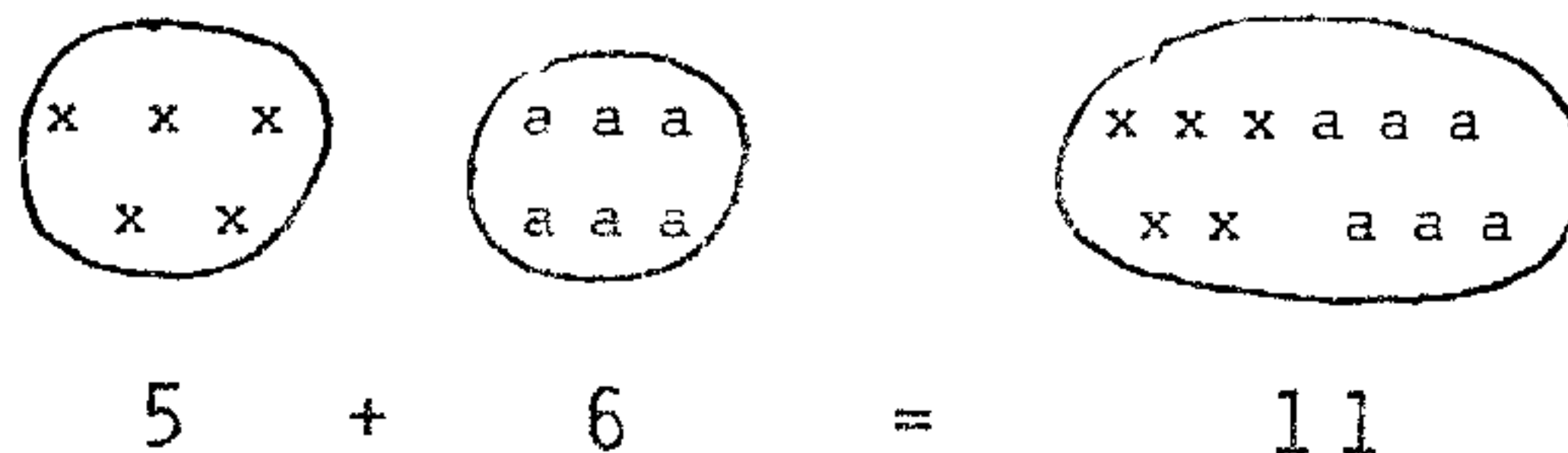
### 3. OPERATIONS WITH NUMBERS

There are many operations that can be carried out with numbers but four are more important than the others. These 'four operations' are ADDITION, SUBTRACTION, MULTIPLICATION and DIVISION.

Other operations which are carried out are FINDING SQUARES, SQUARE ROOTS, FACTORS MULTIPLES, CUBES, LEAST COMMON MULTIPLES, and GREATEST COMMON FACTORS.

### 4. ADDITION

Addition is usually introduced by putting two DISJOINT sets together and in this way addition is associated with the UNION of two sets.



The objects in the first set are counted.

The objects in the second set are counted.

The sets are combined.

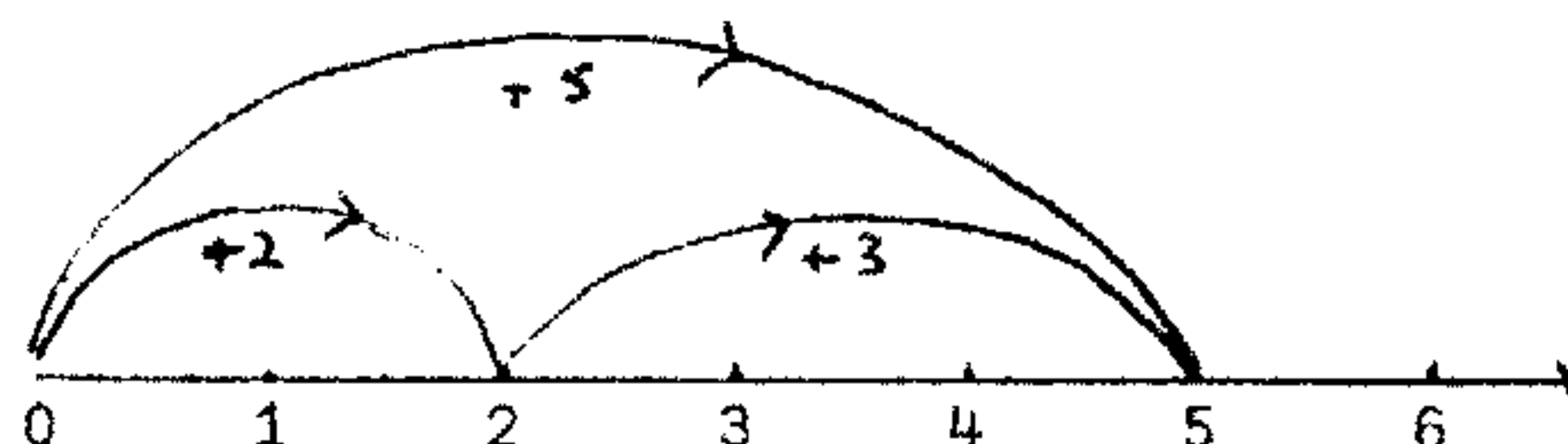
The objects in the combined set are counted.

To add in this way it is necessary to be able to count. Thus, counting systems are needed.

We use A BASE TEN POWER SYSTEM WITH PLACE VALUES and this has to be well understood by all children. This numeration system will not form part of this paper.

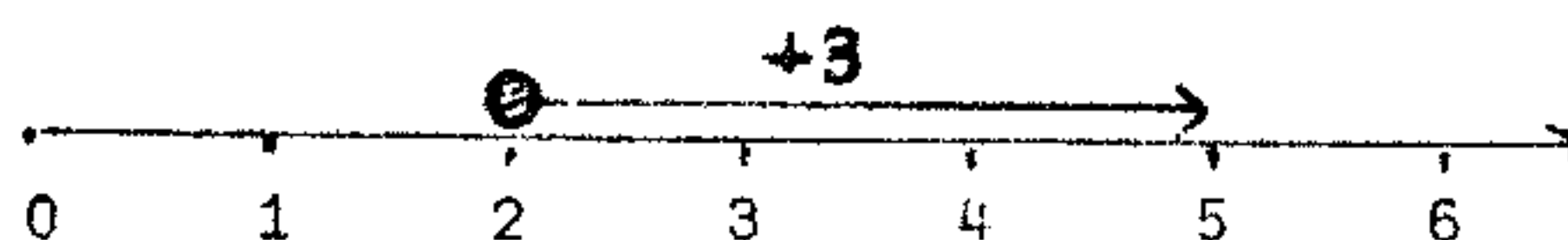
Putting sets together works all right for simple sets of objects but when we get to examples like  $2\frac{1}{2} + 3\frac{3}{8}$  the combining of sets is of no use. The concept of addition must be extended to cover all possibilities.

Addition is often illustrated on a number ray like this:



What is being shown here is combining two jumps. The jump  $+2$  followed by the jump  $+3$  is equivalent to the jump  $+5$ . So  $+2 + +3 = +5$ .

This is not quite the same thing as saying  $2 + 3 = 5$ . Here the 2 and the 5 are STATES and the 3 is an OPERATOR. On a number ray this is shown as



We are making a distinction between THE STATE OF POSITIVE TWO and THE OPERATION OF ADDING TWO. In most countries  $+2$  means ADD TWO and  $^+2$  means POSITIVE TWO as is used in this paper.

Any concept of addition gained by the children must be such that it can be developed into areas like the addition of negative numbers, the addition of matrices and the addition of vectors.

Addition is PUTTING TOGETHER according to some rules. These rules will vary according to the type of mathematical structure being used.

It is important to remember that addition is a binary operation as are all four basic operations. We add two things at a time. No matter how many numbers are to be added we can only add two at a time.

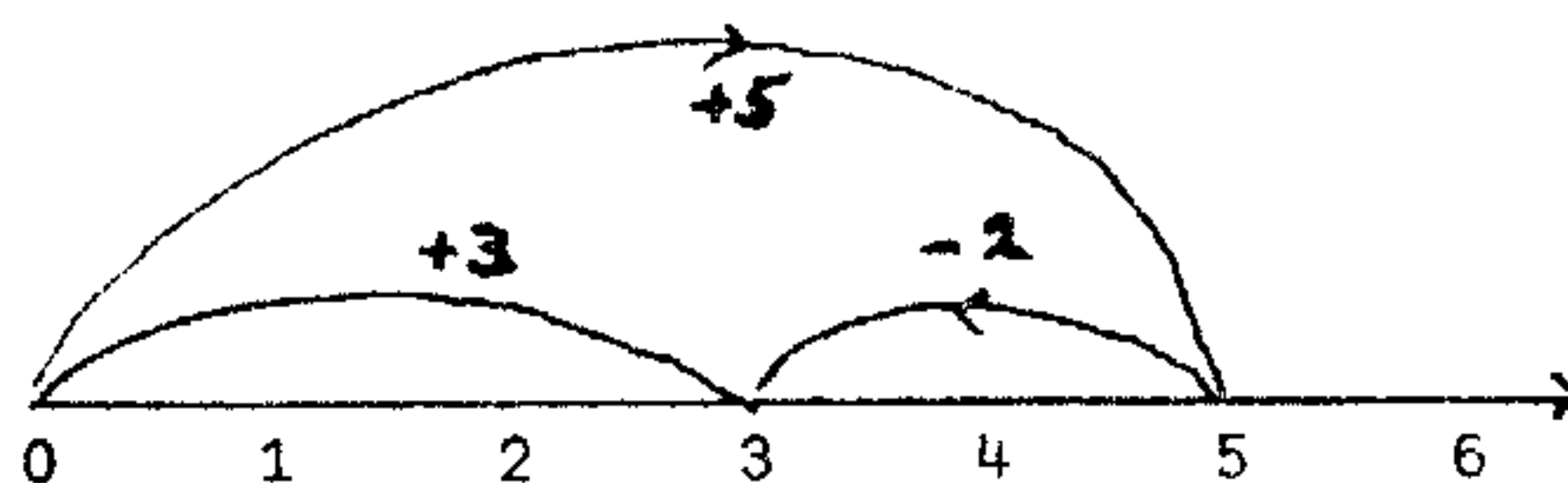
## 5. SUBTRACTION

Subtraction starts by taking objects away from a set and finding how many are left.



In order to preserve the connection between addition and subtraction the set is usually partitioned rather than letting the removed objects disappear.

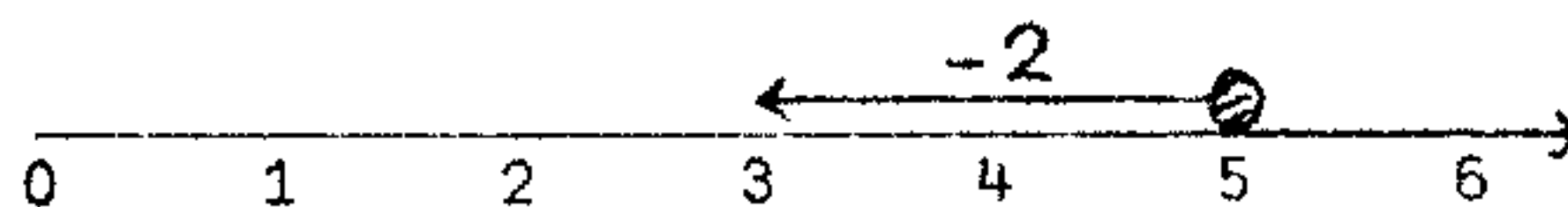
Again this method can only be used for integers so a number ray is used as for addition.



A jump of 5 forward, followed by a jump of two back is equivalent to a jump forward of 3.

This is actually  $+5 - 2 = +3$ .

Since we usually want  $5 - 2 = 3$ , a better representation is

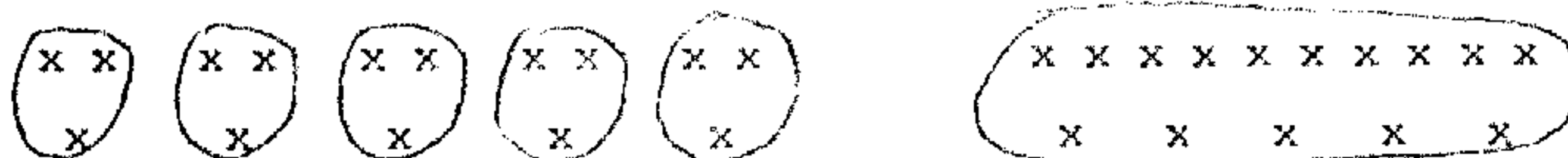


Subtraction is the inverse operation to addition. Adding five can be cancelled by subtracting five.

Subtraction as a separate operation can be avoided by adding negative values. For example,  $5 - 2$  can be done as  $5 + -2$ . However, in a primary school subtraction is dealt with as an operation in its own right.

## 6. MULTIPLICATION

Multiplication is usually introduced as continuous addition. It is the combining of sets with the same number of elements.



3 multiplied by 5 is  $3 + 3 + 3 + 3 + 3$ . The value is found by addition or counting.

There is a problem of language with multiplication. There are two different systems in use.

'What is 5 times 3?' "What is 3 multiplied by 5?"

These are equivalent and imply  $3 + 3 + 3 + 3 + 3$ .

If we want  $5 + 5 + 5$  we say '3 times 5' or "5 multiplied by 3".

This confusion in language is reflected in the symbols for the multiplication.

$5 \times 3$ . Is this '5 times 3' or "3 times 5"? The second seems preferable as normally the operator is written second as in  $5 + 3$ ,  $5 - 3$  and  $5 \div 3$ .

If that is so, it is best to read " $5 \times 3$ " as "5 multiplied by 3" and NOT "5 times 3".

Multiplication is extended to fractions by considering fractions of wholes.

$$\frac{1}{3} \times 6 = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{6}{3} = 2$$

Since multiplication is commutative  $6 \times \frac{1}{3} = \frac{6}{3} = 2$

Following the pattern  $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$  etc.

## 7. DIVISION

Division is introduced in one of two ways.

'Given a certain number of objects to fit evenly into  
a given number of sets'

OR

'Given a fixed number of objects make up sets of a  
given size'

An example of the first is "I have 100 sweets to divide  
out among 20 children"

An example of the second is "I have 100 sweets and I want  
to give 5 to each child"

The first use of the concept of division is to be preferred  
but both are used and must be known.

Division is done as the inverse operation to multiplication  
and the concept of division as an operation in its own right  
is avoided.

It can be done as continuous subtraction. "How many times  
can I take 3 from 12?" is equivalent to "What is 12 divided  
by 3?"

Division treated as inverse multiplication requires statements  
like  $6 \div 3 = 6 \times \frac{1}{3}$  because the multiplicative inverse of 3 is  $\frac{1}{3}$ .  
"turning upside down and multiplying" must be understood by the  
children.

This is best done by a discussion of inverses although the  
following method is possible:

$$\frac{1}{4} \div \frac{1}{3} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{\frac{1}{4} \times \frac{3}{1}}{\frac{1}{3} \times \frac{3}{1}} = \frac{\frac{3}{4}}{1} = \frac{3}{4}$$

### 3. OPERATIONS AND OPERATORS

The four operations have been discussed above. In an expression like  $5 + 3$ , 5 is the initial state, the operation is addition and the operator is  $+$ 5.

The operator has an operation (addition) and a magnitude ( $+$ 5)

When an operator acts on a state we get a final state. In the above it was 8.

It is also possible for an operator to act on another operator. We can have  $+ 5 + 3 = + 8$ .

The operator  $+ 3$  changes the operator  $+ 5$  to the operator  $+ 8$ .

### 9. MATHEMATICAL SYSTEMS

A system is set up by having some set of numbers and some operations that are carried out on the numbers.

Some of the systems that are studied by the children in the first five classes are:

NATURAL NUMBERS WITH ADDITION

NATURAL NUMBERS WITH SUBTRACTION

NATURAL NUMBERS WITH MULTIPLICATION

NATURAL NUMBERS WITH DIVISION

NATURAL NUMBERS WITH ADDITION AND MULTIPLICATION

ALL THE ABOVE WITH WHOLE NUMBERS

ALL THE ABOVE WITH RATIONAL NUMBERS

Mathematical systems are governed by laws and we have to find out which laws apply to which systems.

The laws are:

THE LAW OF CLOSURE

THE LAW OF COMMUTATIVITY

THE LAW OF ASSOCIATIVITY

THE LAW OF DISTRIBUTIVITY



We also need to investigate:

THE EXISTENCE OF IDENTITY ELEMENTS

THE EXISTENCE OF INVERSES

A system of one set of numbers and one operation can have all of these except distributivity as this law requires two operations.

## 10. CLOSURE

If a system is closed any operation will produce a number which is a member of the given set.

Think of the natural numbers with addition. If you add any two natural numbers you will get a natural number. The system is closed.

How about natural numbers with subtraction? The difference of two natural numbers can be positive or negative.

$$8 - 5 = 3 \text{ but } 8 - 12 = -4$$

This system is not closed because negative integers, like  $-4$ , are not natural numbers.

Systems with a zero element and division need care. DIVISION BY ZERO IS BANNED.

## 11. THE COMMUTATIVE LAW

If a system is commutative  $a * b = b * a$ .

The natural numbers with addition is commutative.

For example,  $3 + 5 = 5 + 3$ .

The natural numbers with subtraction is not commutative.

For example,  $5 - 3 \neq 3 - 5$ .

In general, addition and subtraction are commutative but subtraction and division are not with the common number systems.

## 12. THE ASSOCIATIVE LAW

Quite often three numbers have to be combined. For example,  
 $8 + 3 + 4$ .

There are two ways of doing this:

$$8 + 3 + 4 = 11 + 4 = 15$$

$$8 + 3 + 4 = 8 + 7 = 15$$

The two paths gave the same result. Will this always happen with natural numbers and addition? One example cannot prove a theorem although one counter-example can disprove it.

All we can do here is to try a variety of numbers and see what happens. We are soon convinced that no counter-example exists. We cannot formally prove it though.

This intuitive inductive reasoning is quite valid and is the kind the children will be using. It is equivalent to saying "In the light of my experience . . ." It is not the last word and may have to be altered when your experience widens but for you it is true for the moment.

The associative law is:  $(a * b) * c = a * (b * c)$ .

The natural numbers with subtraction is not associative.

For example, try  $9 - 6 - 2$ .

$$9 - 6 - 2 = 3 - 2 = 1$$

$$9 - 6 - 2 = 9 - 4 = 5$$

There are two different answers so the law does not hold. This is a counter-example.

Which of the above answers was CORRECT, 1 or 5? Neither. They are equally correct.

However, we do not like alternative answers so we set up convention so that everyone will agree which answer to accept. The convention we use is: IF ALL THE OPERATIONS ARE THE SAME WORK FROM LEFT TO RIGHT. In other words we accept 1 as the answer.

From this we see a rule that we apply in Algebra. How can  
 $9 - 6 - 2$  always give us 1? When we put a bracket round  
 $-6 - 2$  we CHANGE THE SIGN to  $-(6 + 2)$   
 $9 - 6 - 2 = 9 - (6 + 2) = 9 - 8 = 1.$

### 13. THE DISTRIBUTIVE LAW

The distributive law requires a set of numbers and two operations.  
 Let us take as our example the natural numbers with addition  
 and multiplication.

$6 \times 7 + 8$  without brackets would be done as:

$$6 \times 7 + 8 = 42 + 8 = 50.$$

The convention here is the one known as BODMAS. The order of  
 operations is:

B     brackets sorted out first  
 O     'of' operations  
 D     division  
 M     multiplication  
 A     addition  
 S     subtraction

$6 \times 7 + 8$  has multiplication and addition only so the  
 multiplication is done first.

However, with the distributive law a bracket is in the  
 expression.

$$6 \times (7 + 8). \text{ The answer now is } 6 \times 15 = 90$$

If the system is distributive  $(6 \times 7) + (6 \times 8)$  will also  
 equal 90. It does as  $42 + 48 = 90.$

$$6 \times (7 + 8) = (6 \times 7) + (6 \times 8)$$

Thus the system of natural numbers with addition and multiplication  
 is probably distributive for multiplication over addition.

How about the other way round? Does  $6 + (7 \times 8) = (6 + 7) \times (6 + 8)$  ?

$$6 + (7 \times 8) = 6 + 56 = 62$$

$$(6 + 7)(6 + 8) = 13 \times 14 = 182.$$

The system is not distributive for addition over multiplication.

The general rule for the distributive law for  $*$  over  $@$  is:

$$a * (b @ c) = (a * b) @ (a * c).$$

This is met most often in Algebra with expressions like:

$$3(x + y) = 3x + 3y.$$

## 14. IDENTITIES

An identity element is an operator that has no effect on a state. For example,  $6 + 0 = 6$ . Thus, 0 is the identity element for addition.

To be precise the identity element has no effect whether it comes before or after.  $6 + 0 = 6$  and  $0 + 6 = 6$ .

The general rule is, I is an identity element when  $a * I = I * a = a$ .

## 15. INVERSES

There are two different kinds of inverses; inverse operations and inverse elements.

Subtraction is the inverse operation to addition.

Division is the inverse operation to multiplication.

The inverse of adding 2 is to add 2.

The inverse of dividing by 3 is multiplying by 3.

In these cases we used the inverse operation but what if we only have one operation.

Let us take as our example, integers with addition.

$5 + 2 = 7$ . Starting with 7 what can we ADD to get 5

$7 + x = 5$ .  $x$  has to be  $-2$ .

Thus the inverse element to  $+2$  was  $-2$ .

$3 \times 4 = 12$ . Starting with 12 with what can we MULTIPLY to get 3.

$12 \times a = 3$ .  $a$  has to be  $\frac{1}{4}$ .

The inverse element to 4 is  $\frac{1}{4}$  in this system.

A system with multiplication and a zero element has to be treated with care.

$$a \times 0 = 0.$$

$0 \times b = a$ . This cannot be done as  $0 \times b$  is always zero.

Thus zero has no inverse when the operation is multiplication.

## 16. CLASSIFICATION OF SYSTEMS

Systems can be classified according to which of the properties they possess.

A system with a set of numbers and one operation might turn out to be a GROUP.

A system with a set of numbers and two operations might turn out to be a FIELD or a RING.

Such a classification is outwith the needs of this paper but it is quite an easy extension.

Sets need not be sets of numbers. We can have sets of Geometric States and Transformations making up a Mathematical System.

Other number systems such as matrices and vectors also make up systems.

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PROFESSIONAL WORK

1. PREPARE TWO LESSONS FOR CLASS ONE
 - 1.1 ADDING TWO NUMBERS, SUM LESS THAN TEN
 - 1.2 SUBTRACTING TWO NUMBERS, EACH LESS THAN TENTHE LESSONS MUST STRESS GENERAL RULES SUCH THAT YOU CAN ADD EITHER WAY (COMMUTATIVE) BUT THAT YOU CAN ONLY SUBTRACT ONE WAY (NON-COMMUTATIVE)

2. PREPARE FOUR DYNAMIC CHARTS TO ILLUSTRATE

2.1 $7 + 5 = 12$

2.2 $14 - 8 = 6$

2.3 $4 \times 5 = , =$

2.4 $15 - 3 = ' =$

THE CHARTS MUST BE CAPABLE OF BEING CLEARLY SEEN BY A CLASS TWO OF 60 PUPILS. THE CHARTS MUST HAVE SOME MOVING PARTS.

3. PREPARE A NUMBER RAY FOR USE ON A FLANNEL BOARD TO SHOW CERTAIN COMMON FRACTIONS SUCH AS $\frac{2}{4}$, $\frac{4}{5}$ ETC.

4. PREPARE A LESSON FOR CLASS THREE ON THE CONCEPT OF FRACTIONS AS NUMBERS.

5. PREPARE A LESSON PLAN TO SHOW THE DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER DIVISION FOR NATURAL NUMBERS.

6. PREPARE A LESSON ON THE DIFFERENCE BETWEEN PLUS AND POSTIVE AND MINUS AND NEGATIVE